Preface

This note is designed to be an introduction to spectral properties of the Laplace operators on non-compact asymptotically hyperbolic manifolds and related inverse problems. It started almost ten years ago as a private note of the first author aiming at providing basic knowledge of inverse problems on non-compact manifolds by methods as elementary as possible. There are two backgrounds for this attempt. The first one is the development of the study of the forward problems of spectral and scattering theory for Schrödinger operators, and the second one is the emergence of boundary control method, which is closely connected to the the Gel'fand-Levitan theory and now regarded as an only systematic way to recover the Riemannian metric from the spectral data. The second author had an occasion to stay in Tsukuba University in 2007-2008 as an invited professor, and delivered a series of lectures on the boundary control method. It is a happy memory for us to have been able to discuss during this period the spectral theory, inverse problems and the history of development of functional anaysis and mathematical physics, which constitutes the main part of this note. It is also a big pleasure to thank Matti Lassas for fruitful discussions. In fact, we, Isozaki-Kurylev-Lassas, are working together on inverse scattering problems on non-compact manifolds in a more general framework, and we do feel the necessity of an easily accesible expository articles of the underlying facts in functional analysis, partial differential equations and differential geometry. In recent years, the study of inverse spectral theory on non-compact manifolds is attracting much more interest. Viewed from analysis and geometry, the general Riemannian manifold is a vast subject in inverse problems, and it will be wise to restrict ourselves to some subclasses, in which the hyperbolic manifold is an appropriate object since it is the first non-trivial example of non-compact manifold and has been a source of deep ideas in classical mathematics.

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