

**Institute of Mathematical Statistics**  
**LECTURE NOTES—MONOGRAPH SERIES**

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# **Adaptive Statistical Procedures and Related Topics**

Edited by  
John Van Ryzin

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**Shanti S. Gupta, Series Editor**

**Volume 8**

# **Adaptive Statistical Procedures and Related Topics**

*Proceedings of a Symposium in Honor of Herbert Robbins, June  
7-11, 1985, Brookhaven National Laboratory, Upton, New York*

Edited by

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**Institute of Mathematical Statistics  
Hayward, California**

Institute of Mathematical Statistics  
Lecture Notes–Monograph Series  
Series Editor, Shanti S. Gupta, Purdue University

The production of the IMS Lecture Notes–Monograph Series  
is managed by the IMS Business Office: Nicholas P. Jewell,  
Treasurer, and Jose L. Gonzalez, Business Manager.

Library of Congress Catalog Card Number: 86–83371

International Standard Book Number 0–940600–09–9

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Printed in the United States of America

## PREFACE

Herbert Robbins, Higgins Professor of Mathematical Statistics, Columbia University and Senior Mathematician, Brookhaven National Laboratory celebrated his 70th birthday on January 12, 1985. In honor of this occasion and to pay tribute to his pioneering and influential work in statistics and probability, a symposium was held at Brookhaven National Laboratory on June 7-11, 1985. The idea of the symposium entitled, "Adaptive Statistical Procedures and Related Topics" was to cover the broad range of topics in which Professor Robbins has made fundamental contributions. The term adaptive in the title was meant to capture the spirit of his work in the areas of stochastic approximation, empirical Bayes and sequential analysis. All these areas have the common feature that they adapt to approximate some ideal or optimal statistical procedure as observations accumulate in number and/or time. In addition to this a number of invited speakers, many of whom were or are students and/or colleagues of Professor Robbins, choose to speak on a related topic in probability or statistics. Those papers along with their authors are also given in this volume.

It is hoped this volume captures some of the spirit of the symposium and of the broad and important spectrum of research inspired by the genius of Professor Robbins especially in so many areas of mathematical statistics and probability. However, one thing this volume cannot capture is the actual excitement of the symposium. This includes the warm sharing between Professor Robbins and his many colleagues, friends and intellectual descendants who attended the conference. Also, the volume does not capture the interchange between the participants, their comments and remarks - especially those of Professor Robbins who made many of his usual insightful and often humorous comments.

During the symposium two social occasions of note took place. One on Saturday night, June 8, 1985 was an open house at the home of Herbert and Carol Robbins in East Setauket, New York. The second was a banquet on Monday night,

June 10, 1985 at Berkner Hall at Brookhaven National Laboratory. At this banquet, Dr. George H. Vineyard, Former Director of Brookhaven National Laboratory and a Senior Physicist there, gave a warm and entertaining account of his acquaintanceship with Professor Robbins over their twenty year affiliation at Brookhaven. Also, presented to Professor Robbins at this banquet was a leather bound copy of the book Herbert Robbins: Selected Papers edited by T. L. Lai and D. Siegmund, Springer-Verlag, New York, 1985. Thus he was presented this surprise copy just before it was announced to the public by the publisher. It provides an invaluable resource on the works of Professor Robbins.

The breadth of the ideas of Professor Robbins are illustrated in this Symposium Volume. This volume includes the 36 invited papers from the meeting listed under the five headings.

## I. Sequential Analysis

The ten papers presented in this section make various contributions to sequential analysis an area in which Professor Robbins has made and continues to make essential contributions spanning a forty year period.

## II. Empirical Bayes Theory and Methods

Professor Robbins began the field of empirical Bayes statistical theory with his seminal papers "Asymptotically subminimax solutions of compound statistical decision problems," Proc. Second Berkeley Symp. Math. Statist. Prob., 1, 131-148, 1951; and, "An empirical Bayes approach to statistics," Proc. Third Berkeley Symp. Math. Statist. Prob., 1, 157-163, 1956. Thus, the nine papers on this subject in this volume are direct descendants of these pioneering ideas.

### III. Stochastic Approximation Procedures

Another field started by Professor Robbins in his paper with S. Monro "A stochastic approximation method", Ann. Math. Statist., 22, 400-407, 1951, is the now widely studied and applied field of stochastic approximation. Electrical engineers in particular have found this to be an important statistical methodology. The six papers presented here show the direction of current work in the area.

### IV. Related Topics: Statistics

Those familiar with Professor Robbins' work immediately realize the motivating idea behind most of his work is the solving of important and difficult statistical inference problems. Thus, the six papers in related areas of statistics in this section including the one jointly authored by Robbins are in this spirit.

### V. Related Topics: Probability

The final five papers are on topics in probability theory. Herbert Robbins has throughout his career made both novel uses of and contributions to probability theory. Thus it is most appropriate that the last five papers of the volume reflect this long standing interest of his.

Of the 36 invited papers, 35 were presented at the symposium. Professor Kallianpur was unable to attend but his contribution is included in the volume. Five of the 36 papers presented at the symposium appear at the end of their respective sections in abstract form as they are published elsewhere.

## ACKNOWLEDGEMENTS

The Editor would like to thank all those who attended the symposium. The symposium was attended by approximately 120 persons. Thanks also go to the organizing committee which besides myself included Shanti Gupta, Purdue University, T. L. Lai, Columbia University, Ronald Peierls, Brookhaven National Laboratory, and David Siegmund, Stanford University. Special thanks go to the 30 or so people who refereed and/or read various versions of the manuscripts. In particular, I would like to thank Wei-Yann Tsai, Brookhaven National Laboratory for his help in proofing and reading various manuscripts. Also, I would especially like to thank Paula Appling and JoAnn Langan for the excellent typing. As many of the attendees and authors know the symposium and the production of this volume ran smoothly due very largely to the hard work and cheerful attitude of JoAnn Langan.

In particular, I would like to thank the National Science Foundation, the Army Research Office, the Air Force Office of Scientific Research and the Department of Energy. Without the financial support of these agencies the symposium could not have been held. Also, I wish to thank Brookhaven National Laboratory and the members of its staff who helped provide a wonderful and pleasant atmosphere for the symposium.

Finally, I would like to thank Professor Robbins himself for being a constant source of intellectual stimulation to me for the past 25 years. It gives me great pleasure to work on this symposium in his honor. I treasure him not only as a colleague but as a close and dear friend.

John Van Ryzin  
Columbia University and  
Brookhaven National Laboratory



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