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# **Group Representations in Probability and Statistics**

**Persi Diaconis**

*Harvard University*



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