Completeness has to be restricted: Gödel's interpretation of the parameter t *

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> ... even such successful theories as relativity and quantum mechanics have proven to be vulnerable, for, implicit in each is a latent completeness postulate. Once one realizes that a physical theory should remain open and therefore incomplete, the role of the associated mathematics becomes a crucial issue.
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> W. Yourgrau[1]

In 1949 Gödel presented Einstein with a new solution of the field equations of the general theory of relativity: an exact solution which allows for the bizarre possibility of time-travel.[2] This discovery of a universe with a strange time-structure which is consistent with the general theory of relativity did not however surprise Einstein, or so at least he leads the reader to believe. In his reply to Gödel's contribution, Einstein intimated that the strange feature of a possible closed time-like lines in which the distinction "earlier-later" should be abandoned, "disturbed... [him] already at the time of the building up of the general theory of relativity, without... [him] having succeeded in clarifying it." The weird result of Gödel constituted in Einstein's view "an important contribution to the general theory of relativity, especially to the analysis of the concept of time."[3]

Unlike the general theory, the special theory of relativity has been accepted by the mid-century as an undisputed part of theoretical physics. It has been commonly presented as the invariance theory of the Maxwell equations under the Lorentz group of transformations. However, Einstein seems to have conceived of the special theory as the theory of a special type of gravitational field: the uniform one. This recognition motivated him to go further and require generalization by the inclusion of gravitational phenomena.[4] In

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* This paper is in its final form and no similar paper has been or is being submitted elsewhere.

[1] Yourgrau, 1969, p.80.

[2] Gödel, 1949b.

[3] Schilpp, 1970, p.687.

[4] Kerszberg, 1989, pp.71–72.

this move one wishes that the "causal" structure of space-time in general relativity should manifest locally the same qualities as the flat space-time of special relativity, though globally significant differences are expected to emerge. However, the local nature of general relativity which deals with gravitational phenomena did not combine easily with the global nature of the invariance principles of the electrodynamics of moving bodies. Thus, in Einstein's 1917 cosmological solution of the field equations of his general theory time is conceived of as linear, separated from the three-dimensional structure of space. The solution presents a quasi-absolute time and a preferred coordinate system. Gödel challenged precisely this conception of time — a conception of time which does not cohere satisfactorily with the results of special relativity.

Gödel underlined the "surprising insights into the nature of time" which the special theory of relativity has revealed. "The very starting point of special relativity theory consists in the discovery of a new and very astonishing property of time, namely the relativity of simultaneity, which to a large extent implies that of succession." Simultaneity loses its objective meaning. Special relativity provides, in Gödel's words, "an unequivocal proof" for a philosophical position which "den[ies] the objectivity of change and consider[s] change an illusion or an appearance due to our special mode of perception."[5] In his first paper on this subject, Gödel observed that,

> all cosmological solutions with non-vanishing density of matter ... have the common property that... they contain an "absolute" time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter.[6]

He pointed out that in general relativity,

> the existence of matter,... as well as the particular kind of curvature of space-time produced by it, largely destroy the equivalence of different observers and distinguish some of them conspicuously from the rest, namely, those which follow in their motion the mean motion of matter.

He then continued to make the incisive remark that

> in all cosmological solutions of the gravitational equations... the local times of all *these* observers fit together into one world time, so that apparently it becomes possible to consider this time as the "true" one, which lapses objectively.[7]

Gödel objected strongly to this notion of world, or "true", time which comprises local times associated with the class of privileged observers.

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[5] Gödel, 1949b, p.557.

[6] Gödel, 1949a. See Gödel, 1990, p.190.

[7] Gödel, 1949b, p.559 (emphasis in the original). *Cf.*, North, 1965, p.359.

To retain consistently the relativity of simultaneity, one has to show that there are solutions in the general theory, that is, universes, in which there cannot be such a "world-time", such a "true" time. In these universes the experienced "lapse of time can exist without an objective lapse of time," so that "no reason can be given why an objective lapse of time should be assumed at all." [8] Put differently, it would be wrong, according to Gödel, to believe that the postulated equivalence of inertial observers also enables us to recover equivalence with regard to a unique, absolute time function. The special theory does not allow us to force becoming into any space-time background.

In his solution of the field equations, Gödel sought to implement consistently this result of the special theory: it should be impossible to fit chosen local times into a world-time. By eliminating the Einsteinian "system of three-spaces", he was able to do away with the notion of "absolute" time and recover thereby the result of the special theory of relativity. "It is easily seen," Gödel observed, "that the non-existence of such a system of three- spaces is equivalent with a rotation of matter relative to the compass of inertia."[9]

As *a consequence* of eliminating the three-space system, Gödel obtained a universe in which matter rotates — by all accounts, an absolute *spatial* feature. Matter in the Gödel Universe rotates relative to the path that a test particle follows if it is given an initial radial velocity. One may refer to the tangent of such a path as the "compass of inertia". Thus the compass of inertia rotates relative to the matter of the Gödel Universe, or vice versa.[10] In the Gödel solution rotational mass phenomena occur as a result of the denial that cosmic time would be everywhere orthogonal to the three-dimensional space. By exploiting the geometrization of time, Gödel introduced a limiting case: the parameter t must be interpreted in a certain restrictive sense. Gödel believed he had made the implications of relativity theory, *vis-à-vis* spatialization, inescapable.[11] The existence of closed timelike curves in the Gödel solution deprives time of its unique direction and makes it behave like space, distinguished to be sure from the other three dimensions, but *only geometrically*. Hence the possibility of time-travel: "By making a round trip on a rocket ship in a sufficiently wide curve, it is possible in these worlds," Gödel speculated, "to travel into any region of the past, present, and future,

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[8] Gödel, *ibid.*, p.561.

[9] Gödel, 1990, p.190.

[10] Sklar clarifies this rotation with the following analogy: the path of a particle moving in a straight line out from the center over a phonograph record spinning on a turntable will mark a spiral groove on the record indicating thereby that the record rotates. In the Gödel solution each observer could count himself as central to the spinning of the cosmic matter — an absurdity in the Machian view. Sklar, 1995, p.78. See also Gödel, 1949a; 1950; 1990, pp.190, 212. Cf., Adler *et al.*, 1965, p.377. Hawking and Ellis, 1973, pp.168–70.

[11] Yourgrau, 1991, p.11.

and back again, exactly as it is possible in other worlds to travel to distant parts of space."[12]

It is worth noting that Gödel's interest in this result went beyond the purely theoretical; witness his remarks concerning the physical meaning of the solution and the compilation of astronomical observations. Gödel made a rough estimation of the velocity the rocket ship will have to attain as well as the amount of fuel that would be needed for the journey. The point of these calculations, as Gödel remarked, was to demonstrate that the velocities which would be necessary in order to complete the voyage in a reasonable length of time are far beyond everything that can be expected ever to become a practical possibility; therefore, the result cannot be excluded *a priori* on the ground that the space-time structure of the so-called real world does not allow in principle for such a voyage, after all, the demarcation between difficulties in practice and difficulties in principle is not at all fixed in this situation. Furthermore, there appear to be two notebooks in Gödel's *Nachlass* in which he tabulated angular orientations of galaxies, perhaps in the hope of finding some observational evidence for his solution.[13]

It is important to stress that Gödel did not detect any inconsistency in the notion of cosmic time as expressed originally in Einstein's solution of the field equations. Gödel chose to proceed in a different way than the standard approach. He sought a categorically different solution which was in his view closer to the spirit of relativity.[14] Gödel derived a possible structure of the physical universe from a space-time metric compatible with general relativity, instead of starting from a requirement about this structure.[15] He thereby established a new type of solution of the general theory which indicates in turn a possible existence of a structure of these different types of solution, a structure whose understanding, Gödel believed, could illumine the origin of inertia and the nature of time.

Regarding the further development of the general theory, Gödel did not expect an extension in the sense that the theory should comprehend a broader range of facts, but rather "a mathematical analysis of the equations which would make it possible to take hold of their solutions systematically and to recognize general properties of the solutions." Gödel probably expected that such an analysis of the equations would lead to *some* kind of general theorems about the structure of their solutions, in the sense in which the conservation laws are general theorems about the structure of the solutions of the Newtonian equations: "So far we do not even know the analogue of

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[12] Gödel, 1990, p.205. On time-travel and the spatial sense of time see Malament, 1984, 1987; Pfarr, 1981: the existence of closed timelike world lines in general relativity is not restricted to the peculiar case of Gödel's universe, see, p.1074. *Cf.*, Stein, 1970; Yourgrau, 1991, pp.10–15, 20, 32–38, 43–49.

[13] Gödel, 1990, pp.197–198, 205 fn.11. See, Malament, 1984; 1987, p.2429; Pfarr, 1981, pp.1089–90; Wang, 1991, p.117.

[14] See North, 1965, p.360.

[15] Kerszberg, 1989, p.374.

the fundamental integral formulas of the Newtonian theory," Gödel observed and continued confidently that these integral formulas "must unquestionably exist." He was of the opinion that a deeper physical understanding of the general theory would be obtained if we were to know these integral formulas; or, conversely, if a more precise analysis of the physical content of the theory could lead to such mathematical theorems.[16]

Gödel succeeded in disturbing Einstein's original relativistic cosmology *from within*. While claiming no new principle, he undermined the Mach principle which constituted according to Einstein's initial view one of the three principles of general relativity.[17] According to Grünbaum, "Einstein named his own organic fusion of Riemann's and Mach's ideas 'Mach Principle'." [18] The geometry, or metric relations, of a continuous manifold is determined by forces *extrinsic* to the manifold. Thus, any inertial forces experienced by accelerating bodies and systems must have their origin in an interaction with the material contents of the universe. Indeed, "in a consistent theory of relativity there can be no inertia *relatively to 'space'*, but only an inertia of masses *relatively to one another*." These are Einstein's words. [19] Inertia is associated with the reciprocal interaction of *all* the potentially observable masses in the universe. Einstein's search for a theory that has the feature of completeness comes here to the fore.

This formulation has the immediate implication that the only meaningful idea of motion is motion relative to other material objects, thus the local inertial frame should not rotate with respect to the frame defined by the distant stars; there is no meaning to an intrinsic rotation of the material content of the universe. As it stands, the Gödel solution is inconsistent with this implication of the Mach principle. Gödel's exact solution of Einstein's *modified* field equations of the general theory of relativity presents a completely homogeneous, but not isotropic, *finite* universe filled with matter of constant density (pressure-free perfect fluid) which rotates rigidly relative to the local compass of inertia: the so-called rotating mass solution. The co-moving matter of the Gödel universe undergoes an intrinsic uniform rotation. This result contradicts the Mach principle which expresses the requirement that the bulk matter of the universe determines the compass of inertia, and therefore the two cannot rotate relative to each other. To put it in an equivalent formulation, the demand of the principle that the local inertial compass and the light compass (of the fixed stars) must coincide does not hold in this solution.[20]

The solution implies the disturbing fact that the general theory of relativity may not be after all completely relatavised as Einstein hoped to achieve. Unlike the current popular view, Einstein's general theory of relativity has

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[16] Quoted by Wang, 1991, p.155; *cf.*, p.156.

[17] Einstein, 1918.

[18] Grünbaum, 1974, pp.419–20.

[19] Einstein, 1917, in Lorentz et al., 1952, p.180 (emphases in the original).

[20] Adler *et al.*, 1965, p.377.

not resolved the controversy between the absolute and relative conception of space and time in favour of the latter, at least with respect to the implementation of the Mach principle.[21]

It may seem that there is no connection between Gödel's contribution to logic and mathematics and his interest in the relativity theory. To be sure, the two fields are certainly distinct and the results which Gödel obtained in these fields have different technical impacts.[22] However, these results share a fundamental quality: in both fields Gödel went directly and wholeheartedly to the questions which lie at the very centre of the problems at stake.[23] Thus in logic and mathematics as well as in physics Gödel had a similar intention: to attain a perspicacious view of the very foundations of the fields and to focus on the central, crucial questions. He sought, in a word, foundational problems in an attempt to arrive at definite results on broad conceptual issues. Hence, his interest in the notions of proof and truth and their relation to decidability, and in the notion of time and its relation to physical structure. Such an encompassing approach leads inevitably to the problem of completeness and its mirror image — the limits of knowledge, by all accounts a philosophical issue. Gödel's contributions to logic and mathematics as well as to physics are the fruitful results of a methodology which seeks to delimit the notion of completeness: in logic and in physics, if the sentence and the physical probe, were to be both actor and spectator, both assertion and referent, then the claim to completeness would founder.[24] In a strict sense, Gödel was not a philosopher, but he did see himself as contributing not only to his fields of expertise, but also to philosophy at large. He was concerned with essentially philosophical questions.[25]

Addressing the very foundation of logic and mathematics, Gödel succeeded in forcing a distinction between proof and truth. He arrived at this result by reconsidering the notion of completeness of formal systems. It is only with respect to completeness that the syntactic concepts of derivability and consistency coincide with the semantic concepts of validity and satisfiability. This is not the case when the system is shown to be incomplete. Gödel's "Incompleteness Theorem" is genuinely limitative. He demonstrated that there is an intrinsic limitation on the semantic that can be imposed on the syntactic predicates in Hilbert's formal systems.[26]

Just as the "Incompleteness Theorem" demonstrates in regard to the Hilbert programme that, in that context, mathematical truth cannot be simulated by formal proof, so in regard to Einstein's theory, the construction

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[21] Grünbaum, 1974, p.422.

[22] See Gödel's letter to Seelig (1955) in which he referred to problems in general relativity as "very remote from [his]...own area of work." (Quoted by Wang, 1991, p.155.)

[23] Wang, 1991, pp.2–4, 108–9, 152–54.

[24] Schlegel, 1967, pp.197–98.

[25] Wang, 1991, p.151. *Cf.*, Pais, 1982, p.13.

[26] Yourgrau, 1991, pp.12–13.

of a formal- mathematical model of the Gödel universe demonstrates that, in this context, t cannot be given the intuitive, contentful interpretation of equating the experienced successive, unfolding time with "world-time" or "true" time — an objective temporal becoming.[27] With the derivation of the Gödel Universe as a solution to the field equations of general relativity, Gödel constructed a limiting case for the relativistic geometrization of time. That is, he produced a formal model that essentially limits the possible intuitive, contentful interpretations it can support. As Yourgrau remarks, this construction of limiting cases is "in the great philosophical-mathematical tradition of employing limit concepts to enable the mind to grasp or delimit the most distant reaches of reality."[28]

Gödel's limiting cases originated in a dialectic of the formal and the intuitive. Mathematics for Gödel is not just syntax. Mathematical realism and the expression it finds in Platonism guided Gödel's intuition in logic and mathematics. Gödel explicitly remarked that he did not see any reason why we should have less confidence in mathematical intuition, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them, and, moreover, to believe that a question not decidable now has meaning and may be decided in the future.[29] There are, in Gödel's view, more similarities than differences between sense perceptions and the perceptions of concepts. When one realizes that there are two different sharp concepts mixed together in the original, intuitive concept, then the paradox which has arisen disappears. Similarly, one may not distinguish two neighboring stars a long distance away, but using a high-resolution telescope one can see that there are indeed two stars. This objectivist conception of mathematics marked the entire work of Gödel.[30]

The case of the vicious-circle principle is a good illustration of this point. According to Russell, who gave the principle its original formulation, if an object is defined in terms of some totality, then, if that object were in the totality, one would have a vicious circle. The principle states then that "whatever involves *all* of a collection must not be one of the collection;" or, conversely: "if, provided a certain collection had a total, it would have members definable only in terms of that total, then the said collection has no total."[31] A member of the totality is singled out with the paradoxical property that this member cannot be a member of that totality. Russell intended the principle to rule out as illegitimate and meaningless quantification over the totality of all sets and all statements about the set of all sets. [32] Gödel, on his part, formulated the principle somewhat differently: "no totality can contain members definable only in terms of this totality, or members involving or presupposing this

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[27] *Ibid.*, pp.14–15.

[28] *Ibid.*, p.17.

[29] Gödel, 1990, p.170.

[30] Wang, 1974, pp.8–10, 85, 324.

[31] Wang, 1991, p.317.

[32] Chihara, 1973, pp.4, 6–7. *Cf.*, Grim, 1991, pp.28–31; Priest, 1995, p.150.

totality."[33] It therefore appears that there must exist a definition, that is, a description of construction, which does not refer to the totality to which the object at stake belongs. The construction of an object simply cannot be based on the totality of objects to which the object to be constructed belongs. But Gödel's way out of the circularity is different. It is very characteristic of his approach:

> If, however, it is a question of objects that exist *independently of our constructions*, there is nothing in the least absurd in the existence of totalities containing members which can be described (i.e., uniquely characterized) only by reference to this totality. [34]

This is very striking. Gödel's argument may be regarded, by analogy, as an application of an abstracted version of the Mach principle in mathematics: a member of a totality is referred to uniquely by the totality itself without any resort to an external element (= matter in the universe, the totality, determines the inertial frame at a certain point within the said totality). "It seems to me," Gödel continued,

> that the assumption of such [concrete mathematical] objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions.

Just as physical objects are natural and necessary for organizing our physical experience, mathematical objects are natural and necessary for organizing our mathematical experience. Thus, according to Gödel, if classes and concepts were to be conceived of as real objects, "objects that exist *independently of our constructions*," objects that have been there from the very beginning, then they could have been members of a certain totality, even infinite totality that permits infinite iterations, without falling into the trap of the vicious-circle principle. According to Gödel, "the set-theoretical paradoxes are hardly any more troublesome for mathematics than deceptions of the senses are for physics." He argued that the vicious-circle principle creates such deceptions; it is objectively not true.[35]

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[33] Gödel, 1990, p.125; 1944, pp.123–153. See also Wang, 1991, p.317.

[34] Gödel, 1990, pp.127–28 (emphasis has been added).

[35] Chihara, 1973, p.63. Ramsey pointed out, that "we may refer to a man as the tallest in a group, thus identifying him by means of a totality of which he is himself a member without there being any vicious circle." (Ramsey, 1965, p.41.) Ramsey indeed challenged the validity of the principle itself arguing that to express a proposition indirectly by a reference to a totality "is certainly... a circuitous process, but there is clearly nothing vicious about it." (p.42.) He criticized Whitehead and Russell for using a "rather vague" principle in "a rather slopy way." (pp.76, 24.)

It transpires that a case can be made for the claim that a similar epistemology and ontology guided Gödel's intuition and methodology in the respective fields of logic, mathematics and physics. The Mach principle is a case in point. That Gödel proved its inconsistency with the general theory, does not diminish the force of this principle in inspiring and directing the search for the origin of inertia, the comprehension of time and the construction of cosmological models. After all, the Mach principle is an expression of a belief in completeness, in the totality of objects which exist "independently of our constructions," as Gödel would have it, and determine the course of physical phenomena. It is thus still a moot question whether time, to use Gödel's formulation, "depends on the particular way in which matter and its motion are arranged in the world."[36]

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[36] Gödel, 1949b, p.562. *Cf.*, Stein, 1970, pp.594–95. See also Pais, 1982, p.288.

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