

# THE SPECTRA AND OTHER PROPERTIES OF STARS LYING BELOW THE NORMAL MAIN SEQUENCE

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## 1. Introduction. Stellar evolution

The existence of stars below the main sequence has evolutionary and cosmological significance in various branches of astrophysics. The subluminoous stars, rare in space, compared to late-type dwarfs of the same luminosity, are found in proper motion surveys of faint stars (combined, in the white dwarfs, with color measurements). Few parallaxes or masses are known; since the stars are generally faint, only low dispersion spectra have been obtained till recently. I shall describe some preliminary results obtained in several current investigations on the spectra of subluminoous stars, their location in the Hertzsprung-Russell diagram, their kinematic properties and the evolutionary significance of some of the results.

The following programs on various groups of stars are being carried on with the 200-inch Hale reflector. The high efficiency of the coudé spectrograph, designed by Dr. I. S. Bowen, permits spectroscopy and spectrophotometry conveniently to  $12^m8$  at 18 A/mm and to  $14^m5$  at 38 A/mm. The prime-focus nebular spectrograph will be needed for fainter white dwarfs.

- (A) Spectrophotometry, classification and velocity measurements in the white dwarfs.
- (B) Identification of lines and composition of some O- and B-type subdwarfs.
- (C) Velocities and spectral peculiarities of the B-type "horizontal-branch" stars ( $M = 0$ ) of population II.
- (D) Population II emission-line objects, old novae, SS Cygni stars.
- (E) The subdwarfs of types F and G. Spectral classification, velocity measurements, search for spectroscopic binaries.

It should be pointed out that white dwarfs and F, G and K subdwarfs occur in both of Baade's population types and that some population I B-stars occur somewhat below the normal main sequence in the Scorpio-Centaurus and Orion groups. It is not yet known whether the stars of the two population types can be differentiated spectroscopically in all regions of the H-R diagram.

It is now generally assumed that horizontal-branch stars, subluminoous hot stars, novae and white dwarfs form some type of evolutionary sequence for stars in which the nuclear energy sources are exhausted. The white dwarfs are to be viewed as the

end product of stellar evolution. The mechanical instability and light variation of the SS Cygni stars and of novae at minimum suggest that at least some stars lose matter, and that evolution towards the final white-dwarf stage need not be peaceful. In addition, from Chandrasekhar's theory of completely degenerate stars, the maximum mass of a white dwarf with zero hydrogen content is about  $1.4M_{\odot}$ . Massive stars of population I cannot become white dwarfs without losing mass.

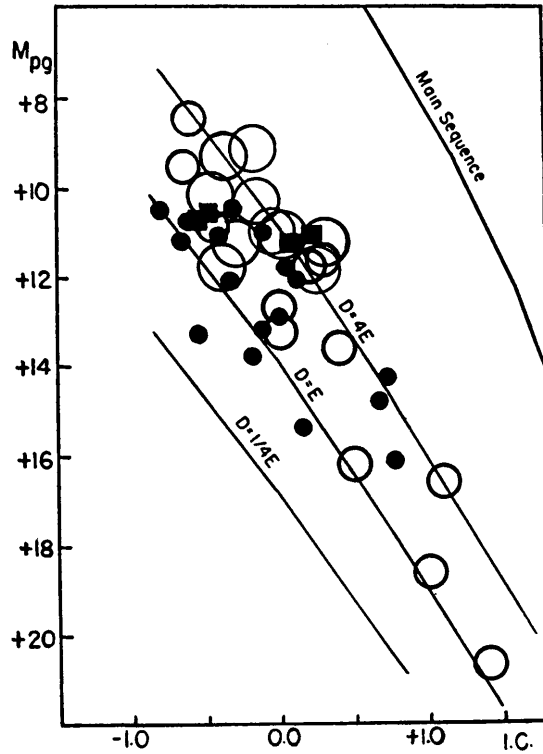


FIGURE 1

The H-R diagram relates color index and luminosity for the white dwarfs (from Luyten). Open circles, uncertain data. The lines give the diameter,  $D$ , in units of that of the earth. Note that a curve parallel to these lines will represent the relation  $M = \text{constant} - 10 \log T$ , the track of a star cooling at constant radius. (Reproduced through the courtesy of Dr. W. J. Luyten, *The Astrophysical Journal*, and The University of Chicago Press.)

Among the oldest main-sequence stars of population II, such as those in globular clusters, the maximum masses are about  $1.25M_{\odot}$ , so their future evolution may be placid. But one billion years ago, and earlier, the globular clusters, and the field of population II stars, contained many objects of mass greater than  $1.4M_{\odot}$ , which have since been forced to lose mass. Only those population II stars recently (if there are any such) formed with large mass, may now be losing mass. A greater rate of formation of hot subdwarfs and white dwarfs via some type of explosive stage probably existed in the recent past. If temperatures in the interiors of such old and formerly massive objects become high enough for heavy-element formation by nuclear processes, a considerable fraction of the total mass of the old stars would have been

ejected into space; the ratios both of helium and of the metals to hydrogen would increase secularly. Such a general picture fits well with the current view that the very old stars of population II are poor in metals compared to recently formed population I or II stars.

## 2. The white dwarfs

Several hundred white dwarfs are known or suspected. For approximately fifty we have some indication of luminosity, color, or spectral characteristics. The most recent discussion of the luminosity-color index relation is by W. J. Luyten [1]. Figure 1, from his paper, gives the H-R diagram as derived from his approximate color indices, and from parallaxes and cluster membership. Unpublished accurate photoelectric colors (measured by D. L. Harris III) exists for many white dwarfs, so that figure 1 can soon be much improved, especially as additional parallaxes are obtained.

Important advances have recently been made in the theory of white dwarfs. In brief resumé, no significant fraction of their mass is hydrogen [2], nuclear energy sources are trivial [3], [4] and they are probably simply cooling [5] by radiation into space, at nearly constant radius. The latter hypothesis, due to Mestel, fits Luyten's observed color-magnitude diagram surprisingly well. Any star that becomes a white dwarf starts at the upper left in the diagram, at a point fixed by its mass (which determines the radius) and its initial luminosity. Then, cooling at constant radius, the effective temperature  $T_e$  will drop, and the star will follow a track given by

$$(1) \quad M_{bol} = \text{Constant} - 10 \log_{10} T_e .$$

The time spent in any stage will be inversely proportional to the luminosity, so that a large number of old, faint and cool "white-dwarfs" should exist. That they have not yet been found is probably due to the method of discovery of white dwarfs, a group of *blue* stars of large proper motion. It may be that some of the red and apparently faint proper-motion stars will eventually prove to be intrinsically faint degenerate stars at  $M > +16$ . In Luyten's diagram, figure 1, curves of constant diameter are shown, that is, the predictions of equation (1), and I wish to point out that the white dwarfs show a mean trend parallel to such lines. The scatter in luminosity at a given color, I then interpret as caused by the initial differences in mass and composition. (The abundance of the heavier elements affects the opacity, so that the rate of cooling is sensitive to composition). For a constant composition, location in the H-R diagram uniquely gives the age and the mass. Since only three masses have been actually reported, no test of this hypothesis is now possible.

I have previously reported the existence of shells of ejected helium near the two helium-rich white dwarfs, and of Ca II near the F-type white dwarf, van Maanen 2 [6]. The spectroscopic results so far obtained at Palomar are summarized below. It should be remembered that because of its very high speed, Bowen's 8-inch aplanatic camera has a spectral range limited to 670 Å on one exposure. No data for the red-green region are available.

2.1. *Group A. Normal hydrogen type.* The best known and apparently simplest group proves to be that with strong broad hydrogen lines. Many objects are known, but spectrophotometric profiles for H $\gamma$ , H $\delta$ , H $\epsilon$ , H $\zeta$  have been measured for the

following<sup>1</sup>: 40 Eri B, W 1346, He 3, SA 29-130, Grw + 70°5824, L 711-10. The hydrogen lines are deep, reaching 50 per cent central absorption, with a width at half central depth of 40 Å for H $\delta$ . Their wings stretch at least 150 Å and overlap for H $\gamma$  and H $\delta$ . Several excellent plates at 18 Å/mm fail to reveal any other lines, sharp or diffuse, in 40 Eri B or Wolf 1346. The hydrogen lines may have relatively sharp cores, but no evidence for a shell or ejection of gas exists. The mean  $M_{pg}$  for this group is +10<sup>m</sup>4, and the mean color -0<sup>m</sup>48, from Luyten's measures. (It should be noted that this color scale deviates appreciably in scale and zero point from the modern B-V photoelectric systems). This group probably contains new white dwarfs of a considerable range in mass. The high abundance of hydrogen in the reversing layer must be explained by the gravitational separation of light and heavy elements, and the possible incomplete exhaustion of hydrogen in the outer envelope [7], [2].

A test of the gravitational red shift, which would be about +20 km/sec for these objects, has been carried out by Popper [8] for 40 Eri B, and I am planning to investigate Sirius B in the next few years, as the separation increases.

2.2. *Group B. Shallow lines.* Several white dwarfs have been found to show extremely broad and shallow hydrogen lines. An appreciable fraction of those hitherto classified as "continuous" will probably fall in this group, for example, HZ 29 (Mal. 132), HZ 43 (L 1409-4) and LDS 678A. In HZ 43, H $\delta$  has a central absorption of 14 per cent, but a half width of 25 Å. Such weak, broad lines require high pressure and either low hydrogen abundance or very high surface temperature. The latter seems probable; HZ 43 is very blue, and has a red companion, probably physically connected; the system is remarkably similar to LDS 678A, B. Such stars should have small radii, large masses and red shifts. Because of the shallowness of the lines, unfortunately, red-shift measures seem impossible. The mean absolute magnitude for HZ 43 and LDS 678A is probably near +10<sup>m</sup>, the mean color -0<sup>m</sup>5. No luminosity data exist for HZ 29, which is a 13<sup>m</sup>6 star of rather small proper motion, but clearly a white dwarf.

2.3. *Group C.* Apparently at higher temperature, and more peculiar than groups A and B of hydrogen white dwarfs, is the star +28°4211. This object is either an extreme subdwarf, or the most luminous white dwarf, somewhere between +3 and +6  $M_{pg}$ . It is the bluest star now known, from photoelectric measures, and its spectrum indicates an extremely high temperature. The hydrogen lines are broad and weak, and the He II line  $\lambda$ 4686 is very strong and relatively sharp. No other lines are visible at 38 Å/mm or on excellent very broad 18 Å/mm spectra, but microphotometer tracings show broad He II lines at  $\lambda$ 4541, 4200, with central depths of only 5 per cent. This spectrum is quite different from other hot subdwarfs, so that classification as a white-dwarf O star seems reasonable. A parallax is badly needed. It may be that some members of group B will also show He II lines.

2.4. *Group D. Sharp lines.* Relatively sharp-lined white dwarfs exist, with  $M_{pg} \approx +12$  and Luyten color of +0<sup>m</sup>1. Only two of these stars have so far been studied, L 870-2, and L 532-81, but several more are on the program. The latter star has an *optical* companion; discovery of others in either galactic clusters or binary systems is very important, since these are the only white dwarfs for which velocities could

<sup>1</sup> The notation for identification refers to discovery lists as follows: He = Hertzsprung; HZ = Humason-Zwicky; L = Luyten; LDS = Luyten double star; SA = Selected Area; W = Wolf; V Ma 2 is sometimes called Van Maanen 1 and is also W 28; Mal. = Malmquist.

be measured accurately. The red shift in such sharp-lined stars is important as an indicator of mass, since there is no obvious explanation of how such sharp lines could occur in a high-pressure atmosphere. If one invokes a large radius, to reduce the surface gravity, the mass of the star becomes quite small compared to that of most white dwarfs. The theoretical question then arises as to how a star of such small mass and presumably low original luminosity should have evolved rapidly enough to exhaust its energy sources. Spectroscopically, L 532-81 is hardly distinguishable from a late B star in strength and width of the hydrogen lines.

TABLE I  
EQUIVALENT WIDTHS IN WHITE DWARFS IN ANGSTROM UNITS

	H $\gamma$	H $\delta$	H $\epsilon$	H $\zeta$	H $\eta$	A $_c$ (H $\delta$ )	Remarks	
<i>Group A</i>								
SA 29	39.3	28.8	17.3	—	—	0.65		
Grw +70°	35.2	23.6	16.9	—	—	0.49		
40 Eri B	31.3	26.7	17.0	7.0	3.3	0.57		
L711-10	32.	24.	8.	—	—	0.45	1 plate	
W 1346	27.0	21.7	13.7	6.6	3.0	0.40	H $\beta$ = 34A	
He 3	28.8	20.2	11.7	4.3	2.3	0.46		
<i>Group B</i>								
HZ 43	9.2	5.4	4.6	—	—	0.14		
HZ 29	4.6	5.2	3.8	—	—	0.11		
LDS 678A	2.28	2.26	1.85	—	—	0.07		
<i>Group C</i>								
+28°4211	4.4	3.0	1.7	1.0	0.6	0.16	$\lambda 4686 = 2.4A$	
<i>Group D</i>								
L870-2	5.5	4.7	3.1	—	—	0.40		
L532-81	14.5	13.7	12.1	—	—	0.50		
	H $\gamma$	G band	4227	H $\delta$	H $\epsilon$	K	H $\zeta$	
<i>Composites</i>								
HZ 19	5.0	1.6	0.45	4.7	5.8	3.0	4.3	core in H lines
-11°162	3.2	1.3	0.27	2.9	3.2	1.2	1.7	

2.5. "Composites." A very difficult problem is raised by the existence of blue stars whose spectra show spectroscopic evidences of low temperature. Humason and Zwicky [9] pointed out a few such cases, in which stars selected by color were found to have F or G spectra, for example, HZ 13 and HZ 41; Luyten ([1], p. 286) also notes several. I have since found that HZ 19 has traces of a late-type spectrum, as does -11°162, a very blue star found by Vyssotsky, and L 1363-3, recently announced as a probable white dwarf by Luyten. In these objects, a broad shallow K line of Ca II is seen, and in HZ 19 and -11°162, the G band,  $\lambda 4227$  of Ca I, and weak traces of Fe I lines are visible. Nevertheless,  $\lambda 4686$  of He II has an equivalent width of 1 A in -11°162, like an O star. It is not impossible that a cool, relatively low-pressure envelope may surround some hot white dwarfs (see below), but it is

also possible that HZ 19,  $-11^{\circ}162$  and L 1363-3 may be unresolvable binary systems. The pair 40 Eri BC, with a white dwarf and a fainter M dwarf might be taken as a prototype. For HZ 19 and  $-11^{\circ}162$ , a difference of about 3 mag. at  $\lambda 3933$  is required, to account for the weak shallow K line. Consequently, the K or M dwarf might be detectable in the infrared. Multicolor photometry would be decisive.

2.6. *Helium-rich stars.* A considerable number of apparently helium-rich stars, white dwarfs, and faint subdwarfs have been found. These have absolute magnitudes determined so far only from proper motions. L 930-80 and L 1573-31 are

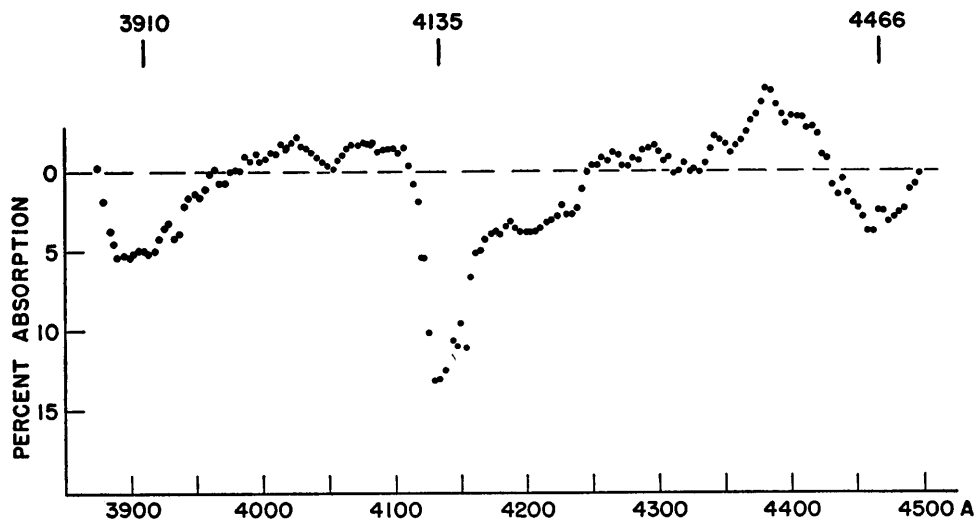


FIGURE 2

The spectrophotometric profiles of diffuse absorption features in the white dwarf Grw  $+70^{\circ}8247$ , a white dwarf classified as of continuous spectrum. The absorption at  $\lambda 4135$  is best determined; the others are of lower weight. A new plate indicates that the peak at  $\lambda 4380$  is not real.

about  $14^m$  visually, and with appreciable motions must have  $M \approx +11$ . Other objects, such as HZ 1, 3, 44 are somewhat brighter, have small motions, and are probably extreme subdwarfs. The stars L 930-80 and L 1573-31 show He I lines stronger than in any other stars known to the author. Table II gives some preliminary results for the subdwarfs and the white dwarf L 1573-31. The data for L 930-80 are poor, but follow the general pattern of L 1573-31, in the presence of cores in certain lines and in the low ratio  $\lambda 4144/\lambda 4121$ . The low ratio may be caused in part by the diffuse and shallow nature of  $\lambda 4144$ , with a resulting loss of measurable equivalent width, and also by a real gain in  $\lambda 4121$ , the  $2^3P^{\circ}-5^3S$  transition. (The leading line of this series,  $\lambda 4713$ , is unfortunately outside the normally observed region.) There is a relatively sharp core in  $\lambda\lambda 4472$  and  $4026$ , and extremely sharp strong cores in  $\lambda\lambda 3965$ ,  $3889$ . The latter two lines have completely metastable lower levels,  $2^1S$  and  $2^3S$ , respectively, and are lines whose relative enhancement is most favored by the dilution of radiation, appearing in absorption in the Orion nebula. It is interesting to note that the two hot stars HZ 1 and HZ 3, which I believe to be hot subdwarfs, also show displaced cores in these two lines. The variety of line profiles is described elsewhere [10].

2.7. "Continuous" spectra. The white dwarfs hitherto classified as having continuous spectra will undoubtedly prove to be a heterogeneous group; most are unfortunately too faint for study with relatively high dispersion. But in at least one star it was already known that unusual absorption features existed. In Grw +70°8247,  $M_{pg} = +12.2$ , photoelectric color,  $B-V = +0^m05$  (that is, redder than a normal white dwarf like W 1346), Minkowski [11] found diffuse bands at  $\lambda\lambda 4135, 4480$ . I have obtained three excellent plates of Grw +70°8247, with results shown in figure 2. The feature at  $\lambda 4135$  is easily seen, but other broad absorptions exist which can only be detected spectrophotometrically. The continuum of a sharp-lined early B star was used as a comparison and the relative intensity at a given wavelength,

TABLE II  
EQUIVALENT WIDTHS IN HOT SUBDWARFS AND A HELIUM-RICH WHITE DWARF

Line	Element	L 1573-31	HZ 1	HZ 44	HZ 3	$\tau$ Sco
4686	He II	—	2.5	1.7	—	0.4
4471	He I	11.7 Sh	4.3	3.0	2.7	1.1
4388	He I	3.6	2.6	2.0	1.8	0.6
4340	H	Abs	0.8	3.8	5.6	3.1
4144	He I	0.9	1.7	1.3	1.1	0.5
4121	He I	1.2	0.5	0.4	0.7	0.3
4116	Si IV	Abs	0.1	0.2	0.2	0.1
4101	H	Abs	0.1	4.1	4.1	3.4
4026	He I	7.7 Sh	3.3	3.3	3.0	1.2
3965	He I	2.8 Sh !	2.1 Sh	0.6	0.2 Sh	0.2
3889	H + He I	4.9 Sh !	— Sh !	2.4 Sh	0.9 Sh !	3.3

Note: A dash means no satisfactory observational material yet exists; "abs" means absent; "Sh" means sharp core, that is, a shell phenomenon in part; "Sh!" means strong shell phenomenon. Results for  $\tau$  Sco are quoted from Unsold.

white dwarf/normal stellar continuum measured. Each plate was measured three times, at arbitrary unsmoothed points, and a mean then taken. From the results of figure 2 it seems probable that broad absorptions exist at  $\lambda\lambda 3910, 4135, 4466$ . The extraordinary width, shallowness and lack of obvious identification make these results extremely puzzling. No known stellar or laboratory spectra have any resemblance to that in figure 2. A most tentative suggestion is that these bands are the result of pressure broadening in the He I spectrum. In that case,  $\lambda 3910$  could be a mixture of  $\lambda 3889$  ( $2^3S-3^3P^0$ ) and  $\lambda 3965$  ( $2^1S-4^1P^0$ );  $\lambda 4135$  would be  $\lambda 4121$  ( $2^3P^0-5^3S$ ) and  $\lambda 4144$  ( $2^1P^0-6^1D$ );  $\lambda 4466$  would be  $\lambda 4438$  ( $2^1P^0-5^1S$ ) and  $\lambda 4472$  ( $2^3P^0-4^3D$ ). High levels of  $n^3D$  would be most, and  $n^1S$  least subject to pressure effects, possibly explaining the absence of  $\lambda 4026$ . An as yet unchecked prediction is that a relatively sharp feature would exist near  $\lambda\lambda 7066$  and  $4713$ ,  $2^3P^0-3^3S$  and  $2^3P^0-4^3S$ . Weak support for identification of the  $\lambda 4135$  dip with  $\lambda 4121$  is that the ratio  $\lambda 4121/4144$  exceeds unity (table II) in L 1573-31 (and also in L 930-80, rough data), different from its value in any other star. The luminosity effects in the singlet-triplet ratio are less relevant here than in normal stars; the damping constants are probably so large that curve-of-growth effects are less important than whether the upper level of an atomic transition continues to exist at high pressures. Levels with the smallest mean atomic radius will persist longest. The lines should shift by an appreciable fraction of their collisional damping constant.

2.8. *F-type spectra.* I have elsewhere described [6], [10] the spectrum of Van Maanen 2 in the ultraviolet. Very broad, overlapping Fe I lines from the lowest energy states exist, and were shown to have a width corresponding to a mean damping constant 60,000 times the classical value. The indicated pressure is about 60 times the expected value based on the mass, radius and opacity. Part of this excess may arise from the effect of deeper layers on line broadening, from the opacity, and part on possible rotation. However, a tentative first theoretical analysis of the observed value gives some interesting results. If I assume quadratic Stark broadening, an electron pressure of  $10^6$  bar is derived from laboratory measures on Fe I. If the electrons come from the metals, the gas/electron pressure is the ratio of the abundances of hydrogen plus helium to the metals. It therefore does not matter too much whether hydrogen is partially or completely converted into helium; the gas pressure will be about  $10^9$  bar. The particle density,  $N = P/kT = 10^{21}/\text{cm}^3$ , gives an interatomic spacing of 10 Å. The lowest energy states of iron probably have an effective radius of 3 Å; highly excited levels will have too large a radius to exist. This has an interesting confirmation in the absence of the strong Fe I  $\lambda 4045$  and  $\lambda 4383$  multiplets. These well-known lines, with 1.5 eV excitation potential, have an effective  $Nf$  value less than some of the ultraviolet lines; because of their freedom from blending, they should have appeared as broad lines at least five per cent deep. However, they are not visible.

In addition to the broad Fe I lines, sharp cores appear superposed on broad Ca II lines. These resemble lines formed in shells, and probably have variable velocity. Recently, I have found that L 745-46A has similar sharp cores superposed on the broad Ca II lines. These two objects have  $M_{pg} \approx +14.5$ ; they are considerably fainter than any of the preceding groups described.

2.9. *Conclusion.* It is clear that a large variety of types of white dwarf exist. It is improbable that there will be narrow sequences in the H-R diagram, or that there will be a simple correlation of spectral characteristics and color. The H/He surface abundance may depend on the balance between gravitational diffusion and convection. In addition, there exists the possibility of heavy-element formation in the early history of the more massive white dwarfs, so that the ratio He/metals may be variable. One of the most striking facts, to me, is that all four white dwarfs with He I or Ca II lines show evidences for shells, that is, ejection of material. In all theoretical work, white dwarfs have been assumed to be mechanically stable. Mestel's work [5] would be seriously affected by mass-loss during the evolution of the older white dwarfs. If many white dwarfs prove to be unstable, they may be viewed as possible sources for heavy elements in the interstellar gas and in the more recently formed stars of population I.

### 3. Horizontal branch stars and hot subdwarfs

Among the faint blue stars found by Humason and Zwicky [9], proper motion studies have already indicated the existence of stars well below the main sequence. Preliminary estimates [12] that the mean absolute magnitude of the entire group was about  $0^m0$  to  $+0^m5$  need elaboration because of the large spread in luminosity indicated by the current spectroscopic study. A considerable fraction of the objects of types B2 or later are probably population II horizontal branch stars; the balance are white dwarfs or hot subdwarfs, that is, considerably fainter. The number of hot



stars below the main sequence must be large, and it is hoped that further surveys for faint blue stars will provide material for a complete statistical study.

3.1. *Horizontal branch stars.* In collaboration with Prof. Guido Münch I have studied the spectra and luminosities of a number of the HZ stars at the galactic pole, and of other blue objects kindly supplied by various observers. If we isolate those of type B2 and later, we have already observed a group of 12 blue stars with a mean  $m_{pg} = 11.8$ . Our observed radial velocities, measured at 38 A/mm, and corrected for the normal solar motion, give an observed dispersion about their mean of  $\pm 48$  km/sec. Dr. M. L. Humason kindly provided lower accuracy unpublished material, which after a similar analysis gave an intrinsic velocity dispersion of  $\pm 54$  km/sec. There is little doubt that the faint blue stars have moderate high-velocity characteristics. Proper motions were available for only 7 of these stars [13], so that only a preliminary determination of the mean luminosity was made by Dr. Münch and myself.

Assume first that these stars show the reflex of the normal solar motion. The mean parallax  $\bar{p}_v$  was derived separately from  $\mu_\alpha$  and from  $\mu_\delta$

$$(2) \quad \begin{aligned} \bar{p}_v &= 0''.00012 \pm 0''.0050 && \text{from } \mu_\alpha, \\ \bar{p}_v &= 0''.0085 \pm 0''.0016 && \text{from } \mu_\delta. \end{aligned}$$

The mean luminosity would then be unreasonably low,  $M = +6$  from  $\mu_\delta$ . The motion is clearly not the normal solar drift. Assume instead that the motion has the direction of the reflex of the solar galactic rotation, at an unknown velocity  $V_\odot$ . Again  $\mu_\alpha$  seems to give incorrect results, but from the mean  $\bar{\mu}_\delta = -0''.0110$ , we derive<sup>2</sup>

$$(3) \quad \bar{M}_{pg} = 11.2 - 5 \log V_\odot,$$

which gives

$V_\odot$ km/sec	$\bar{M}_{pg}$
20	+4.7
50	+2.7
100	+1.2
150	+0.3
200	-0.3

In principle,  $V_\odot$  can be obtained from the observed radial velocity dispersion  $\pm 48$  km/sec, by Stromberg's parabolic relation between the solar motion and the velocity dispersion. The dispersion in space motions, on the assumption of spherical symmetry would be about  $\pm 84$  km/sec, in which case  $V_\odot$  would be 125 km/sec, and  $\bar{M}_{pg} = +0.7$ .

We make a similar analysis using the  $\tau$ -components, correcting for the errors of the measured proper motions. In that case the value of  $\bar{M}_{pg}$  is +1.6, again on the assumption that the tangential velocity dispersion can be derived from radial ve-

<sup>2</sup> Because of the small size of  $\bar{\mu}_\delta$ , compared to the probable error of about  $\pm 0''.007$  per star, we attempted to use all the proper motions of B3 to A0 stars in the HZ list, which were not white dwarfs [13]. The  $\bar{\mu}_\delta$  then found was  $-0''.0106 \pm 0''.0013$ ; since the larger list has a fainter mean apparent magnitude, the predicted  $\bar{\mu}_\delta$  for our seven stars would have been  $-0''.0176$ , and the derived  $\bar{M}_{pg}$  for a given  $V_\odot$  even *fainter* than shown above. We therefore used the actual value observed for the smaller group.

locities using a spherically symmetrical space-velocity distribution. The mean of the two methods, near  $+1^m1$ , is somewhat below that usually assumed for the horizontal-branch stars,  $M = 0^m0$ . The peculiar behavior of  $\mu_\alpha$  and the small number of stars suggest that the present group of 12th mag. B3 to A0 stars, with a mean type B7, has a mean luminosity somewhere between 0.0 and  $+2.0$ , a range which is sufficiently small to establish that the stars are somewhat below the main sequence, on which B7 corresponds to  $M_{pg} = -1.2$ . Thus the later type HZ stars are in part horizontal-branch stars, or fainter.

Examination of the available spectra lead to the following conclusions:

- (1) Stellar rotation is small.
- (2) Lines tend to be slightly weak, for example Mg II.
- (3) At B3, N II may be present, O II and C II not at all.
- (4) The interstellar K line is often seen, with a velocity corresponding to nearby gas (the stars being at an average height of 1000 psc above the galactic plane).

3.2. *Hot subdwarfs.* From proper motions, several of the HZ stars could be subdwarfs or unidentified white dwarfs. HZ 1, HZ 3 and HZ 44 show spectroscopic characteristics which would relate them to the subdwarfs.

The proper motions recently determined for the 13th mag. stars, HZ 1 and HZ 3, are small [14] and disagree with older values [9]. HZ 44, a 10th mag. star, has  $\mu_\alpha = -0^m070$ ,  $\mu_\delta = +0^m020$  [13], that is, much larger than the value found for the horizontal-branch stars. Since it has a low radial velocity,  $-15$  km/sec, it need not be related to the horizontal-branch stars. Its luminosity is difficult to estimate, but is not brighter than  $+3$  (assuming a tangential motion of 125 km/sec), and probably considerably fainter. However, the presence of  $\lambda 4686$  in HZ 1 and HZ 44 indicates a late O type, corresponding to a luminosity of  $-4$  if on the main sequence. What happens at the blue end of the horizontal branch in globular clusters is not yet known but an evolution towards the white dwarfs is probable, and may produce objects like HZ 1, 3 and 44.

A preliminary survey of spectra of HZ 44 at 18 A/mm has been made by Münch and myself,  $\lambda 3400$ - $\lambda 4800$ , and about 400 lines have been measured. The rotation is small. The hydrogen lines are weak, He I and He II strong, N II, N III very strong, O II weak and C II doubtful. Other high-temperature lines such as A II, Ne II, Si III, Si IV, Fe III, etc., are present. This extraordinary spectrum shows strong pressure broadened He I lines (table II); almost certainly the ratio He/H is abnormally high, as is the N/C ratio. The abundance anomalies are those expected in a star in which the hydrogen is nearly exhausted, the carbon-cycle equilibrium ratio N/C established [15], and in which some mixing has occurred at a later epoch.

The stars HZ 1 and HZ 3 are also high-temperature objects showing Si IV and He II, and a large He/H ratio. Since they are 13th mag. stars, and the 38 A/mm dispersion is used, the large numbers of weak lines of HZ 44 are not seen; in addition the pressure broadening seems greater. Since, as previously remarked, they show some of the characteristics of the helium white dwarfs, that is, the sharp cores in the metastable lines, they may represent a further stage in evolution towards the white dwarfs than does HZ 44. Measurement of the parallax of these objects would be very useful. It is clear from the small rotation of these hot objects, which must have small radii, that they cannot be derived by collapse of early-type parents of

the population I main sequence, unless the rotational angular momentum has been greatly reduced. They may be the residues of type II red giants, in which the low-density envelope has been lost, or collapsed, and the degenerate core represents a large fraction of the mass. On the other hand, no evolutionary path has yet been traced for population I stars of large mass after they pass through the red giant or supergiant stage; in such an expanded state, rotational momentum might be lost, with the loss of the envelope.

#### 4. High-velocity dwarfs and subdwarfs of types F and G

4.1. *The spectra.* A large number of high-velocity dwarfs have been found from proper motion and radial velocity surveys. Spectroscopic criteria for recognition of the subdwarfs have never been very clearly stated, although G. P. Kuiper and A. H. Joy have indicated that they used mainly the weakness of the metallic lines. The abundance of the metals may be very low in stars of very high velocity, the "halo population II," as compared to the ordinary, high-velocity "disk population II." The absolute strength of metallic lines has a considerable effect on the spectral type assigned to stars in the range A0 to K0, especially at low dispersion where all metallic lines, except Ca II and Mg II, are usually grouped together. The hydrogen/metal ratio may also affect the level of ionization and the opacity. It would therefore be desirable to develop a spectral classification system which is independent of the abundances of the metals, and subsequently to investigate the dependence of the absolute line strengths, at a given type, on kinematic properties.

At Palomar a dispersion of 18 Å/mm can be used to provide spectra suitable for an excitation-temperature classification of the late A to late G subdwarfs. The resolution permits use of ratios of Mg I, Fe I, Cr I and Ca I lines with excitation potentials ranging from 4 to 0 eV. Unfortunately no single element has lines of sufficient range of excitation to provide convenient line pairs, so that in practice, ratios like Fe I 2.5 eV and Mg I, 4 eV to Ca I and Cr I, 0 eV had to be used. Variations of relative abundance of Fe to Ca and Cr would have a serious effect. It is hoped that any possible changes do not affect the relative abundances of the metals, but mainly the H/metal ratio. It was found that the ratio of the ions Sr II, Fe II, Ti II to the neutral elements was consistent with the temperature classification based on the neutral elements alone, and they were given some weight. No account was taken of the strength of CH, of hydrogen, or of Mg II ( $\lambda 4481$  is almost never visible). It may seem strange that so much difficulty should arise in classifying relatively simple spectra, but all lines are extraordinarily weak in extreme F-type subdwarfs. They are sharp and weak enough so that even at 18 Å/mm only 30 to 50 lines are visible from  $\lambda 3900$  to  $\lambda 4600$ .

Approximately 80 stars are on the observing program, and about 50 have so far been observed at least once. The classification is still in a preliminary stage, and the results are quite uncertain. I ordered the stars in a temperature sequence, and then divided them according to line strength at a given type. There is little uncertainty in connecting this temperature sequence with that of normal stars at about G8 V, but it is quite uncertain whether the stars I call dA8-F0 are of the same temperature as F0 V stars, and in fact I believe that at present I classify the extreme subdwarfs too early at F0. Too few photoelectric colors are as yet available to check my spectral-type-temperature relation. The colors plotted in figure 3 and compared

with the Johnson and Morgan [16] main sequence, are from Miss Roman [17], D. L. Harris III (unpublished), and others. Some stars, not yet observed by me, have classifications by Roman [17], and Joy or Popper as quoted by R. E. Wilson [18]. In addition to my spectral types, I have indicated my estimate of the metallic-line strength for the given spectral type as follows: "weak lines" = wk; "very weak, but intermediate" = id; "extremely weak" = sd. It will be shown that the wk stars are mainly ordinary population II disk stars, although some low-velocity

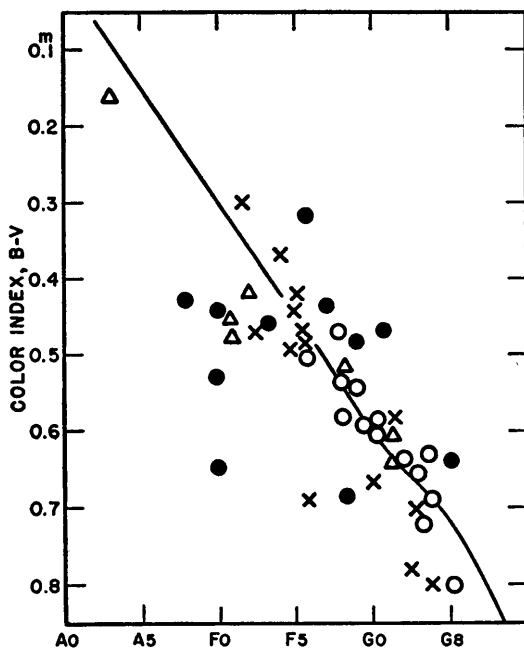


FIGURE 3

The relation between B-V photoelectric colors and spectral type for the Johnson-Morgan main sequence is compared with miscellaneous color data for high velocity A to G stars. Stars classified by others, or uncertain, are indicated by X. The wk stars are open circles, id are triangles, and sd solid dots.

stars may be included, and that the id and sd stars are connected with the halo population II, and differ from each other only slightly in kinematic properties. The luminosities were not known at the time of classification by line strength.

Figure 3 shows that the wk stars here classified have about the same color-spectral type relation as the normal main sequence. The id and sd stars show larger accidental errors, and at least some of the sd stars are obviously classified too early for their colors. The most discrepant point is HD 195636, for which Roman gives a color of  $+0^m65$  and a type G0; I classify it as sdF0, but this early type receives support from the weakness of the CH lines of the G-band. The id stars show less scatter. It is apparent that the problem of spectral classification of stars with extremely weak lines has not yet been solved, and I therefore do not wish to give the data for individual objects at so early a stage in the investigation. The separation between wk and id and sd groups, however, is sufficient to permit an investigation of the luminosities.

4.2. *Luminosities and kinematic properties of subdwarfs.* In a first investigation of the luminosities of the A and early F subdwarfs [19] I made a preliminary estimate of  $M_{pg} = +6.5$  for 25 objects. A reliable determination from trigonometric parallaxes, and from proper motions, can be made when the subdwarfs are correctly separated from ordinary high-velocity stars, and when more parallaxes are measured. The large space motions found by Miss Roman [17] indicate that proper motions can be used only when a good separation is made into kinematically homogeneous groups.

Since the solar motion with respect to the local centroid is small compared to the

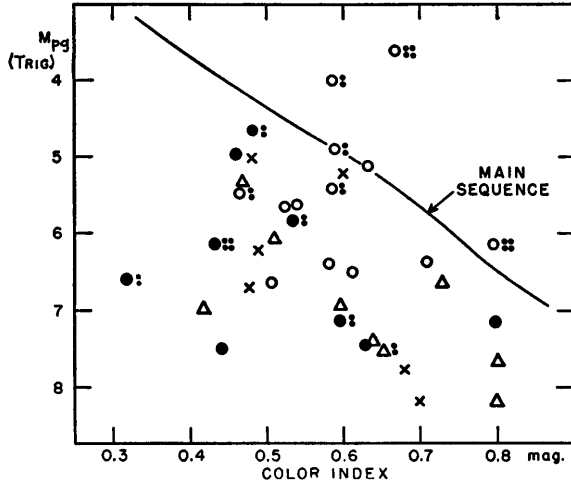


FIGURE 4

The relation between  $M_{pg}$ , the luminosity determined from trigonometric parallaxes, and colors. Colons or double colons represent stars with uncertain parallaxes. The main sequence from Johnson and Morgan.

space motions of these stars, a first approximation to the mean parallax can be obtained from  $p_r$  and from the mean tangential velocity  $V_r$

$$(4) \quad p_r = \frac{4.74\mu}{V_r} ;$$

using the reduced proper motion  $H = m + 5 \log \mu$ , equation (4) becomes

$$(5) \quad M = H + 8.39 - 5 \log V_r .$$

In a statistical application of equation (5), we use the total proper motion  $\mu$  and estimate or compute  $V_r$  from the radial velocities. If  $V$  is the total space motion with respect to the sun, the expected mean value  $\bar{V}_r = \pi V/4$ , and of  $\overline{V_r^2} = \sqrt{2/3} V$ ; the expected mean radial speed is  $\overline{|V_r|} = V/2$  and of  $\overline{V_r^3} = \sqrt{1/3} V$ . There were 43 radial velocities of sd and id stars, either from R. E. Wilson's catalogue [18], probable errors  $\pm 5$  to  $\pm 10$  km/sec, or my unpublished values, probable error  $\pm 1$  km/sec. I found  $\overline{|V_r|} = 140$  km/sec and  $\overline{V_r^3} = 170$  km/sec, a ratio of 0.82, while for a normal error distribution the ratio is 0.798. Since we lack the parameters of the velocity ellipsoid of subdwarfs, I assume spherical symmetry, neglect the solar

motion, and for this first approximation use means instead of root-mean squares in equation (4). Then  $\bar{V}_r/|\xi| = \pi/2$ .

TABLE III  
TANGENTIAL VELOCITIES

Type	Number	$\bar{V}_r$ (km/sec)
sd	19	254
id	12	163
sd + id	31	219
wk	22	57

Figure 5 shows the relation between trigonometric parallaxes and total proper motions, that is, the data for equation (4). All parallaxes, including low weight and negative values are plotted. Six nearby stars are not shown in figure 5 because of

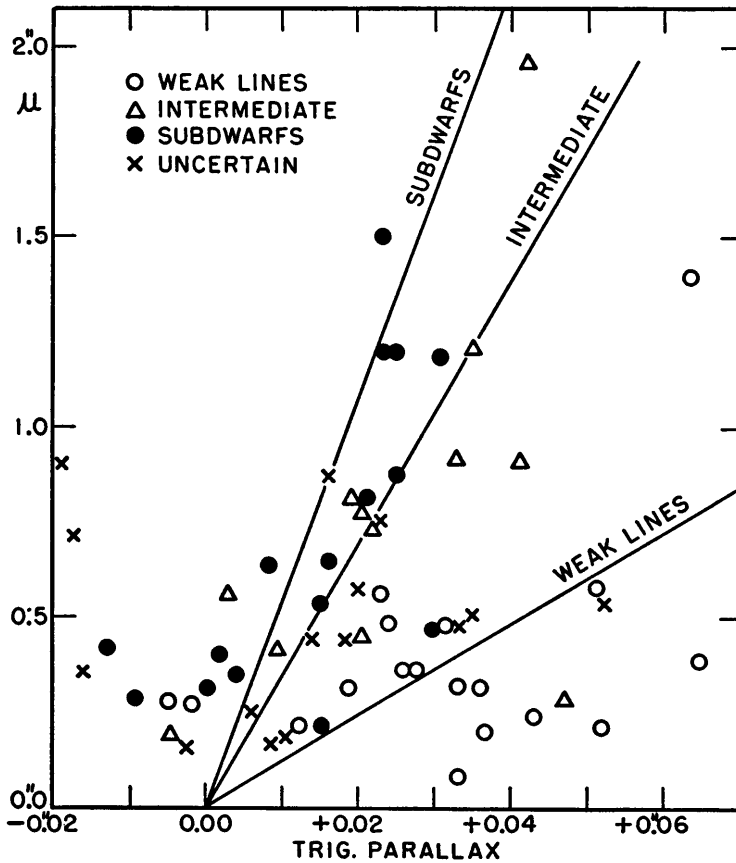


FIGURE 5

The relation between proper motion,  $\mu$ , and trigonometric parallax,  $p$ . Straight lines forced through  $p = \mu = 0$  determine the mean transverse velocity for each group.

their large  $p$  and  $\mu$  which would require a different scale, but were used in the analysis. The best straight line forced through  $\mu = p = 0$  was used to determine  $V_r$  from equation (4) with the results in table III. The observed  $|\bar{\zeta}| = 140$  km/sec for 43 sd and id stars (a larger sample than used in table III, since some parallaxes were missing). The predicted value of  $\bar{V}_r = 220$  km/sec, from radial velocities while

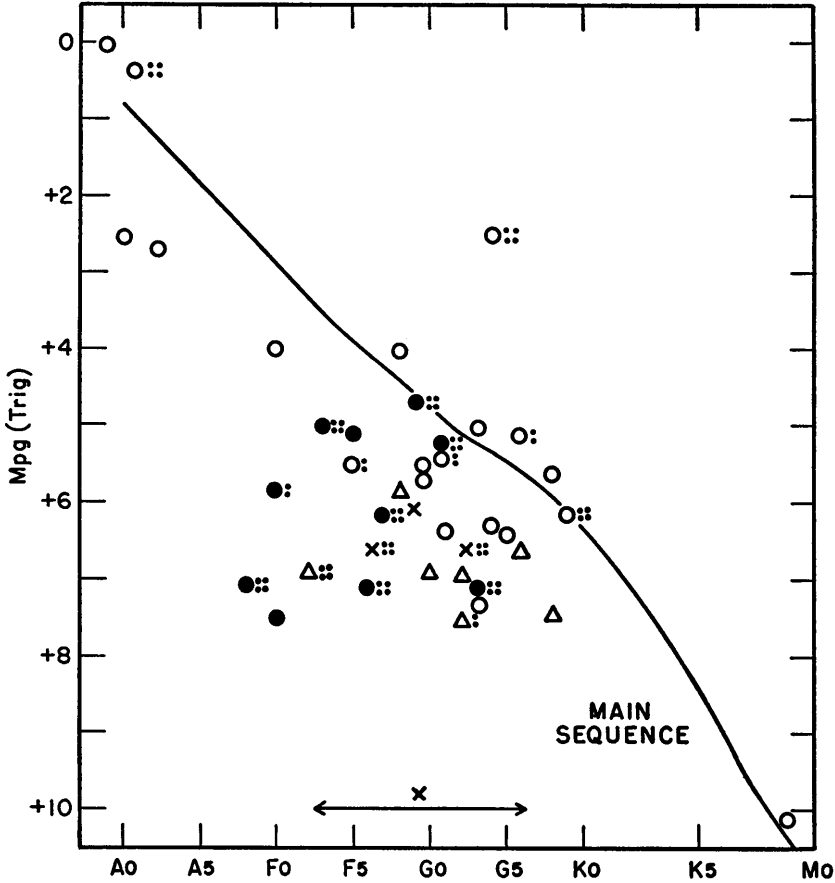


FIGURE 6

The H-R diagram based on trigonometric parallaxes and the present spectral type. Symbols as in figures 3, 4, 5; colons for poor parallaxes. The horizontal arrows refer to a star of improbably low luminosity, +38°9455.

the observed value in table III is 219 km/sec. Then from equation (5) we derive  $\bar{M} = \bar{H} - 3.3$ . The wk stars have such small  $\bar{V}_r$  that this statistical method (uncorrected for solar motion) cannot be used.

The plot of  $H$  against the trigonometric luminosities shows enormous scatter for the 20 stars known to be sd or id from spectroscopic evidence. Only eight stars have  $p > 0''30$ . The result is  $\bar{M} = \bar{H} - 2.5$ , with  $\bar{M}_{pp}$  ranging from +5 to +7. The two determinations of the  $\bar{M}, \bar{H}$  relation are not completely independent, but the mean result is

$$(6) \quad \bar{M} = \bar{H} - 2.9, \quad (\text{sd and id}),$$

disagreeing by about 0.9 mag. from my earlier result [19]. Equation (6) permits luminosity determinations from proper motions only, can be used for more stars, and seems to give somewhat better results than the few available parallaxes. In a few cases where  $\mu$  is accidentally very small,<sup>3</sup>  $H$  cannot be used. Figures 6 and 7 show the results. It can be seen that there is no large deviation of the wk stars from the normal main sequence, and that the sd and id stars are below and to the left.

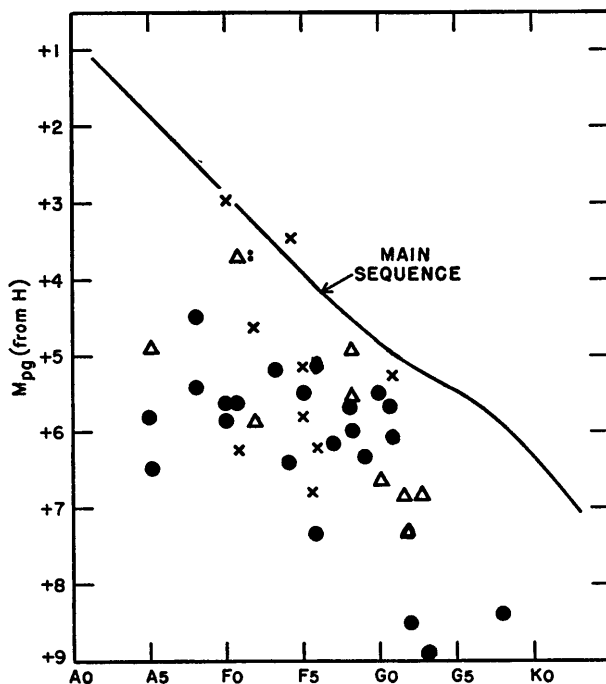


FIGURE 7

The H-R diagram based on proper motions, for the id and sd stars of the present investigation. (X stars classified by others, no line strength data available.) No wk stars are included. The colon is for a star of accidentally small proper motion. The G2 and G8 stars near  $M = +8.5$  have extremely large proper motions; the G8 star is Groombridge 1830, with a trigonometric luminosity  $M_{pg} = +7.4$ .

The mean values of  $M_{pg}$  are given in table IV; there is a larger fraction of id's in the later types. The data for stars earlier than F0 are poor. In the mean, the luminosity of the subdwarfs is about +1.4 mag. less than that on the main sequence for types later than F0; those I classify earlier may deviate slightly more. The gross mean  $M_{pg}$  for the range F0 to G0 is +5.4, or about  $M_v = +5.0$ . In this connection, if we use the value  $M_v = +5$  for the stars where Miss Roman obtained galactic orbits ([17], table III) we obtain what might be viewed as less extreme orbital characteristics. She finds that if  $M = +4$ , out of 13 orbits 7 are retrograde and 5 are hyperbolic; on the other hand, if  $M = +5$ , only 2 are retrograde and 3 are hyper-

<sup>3</sup> For example HD 161817, idA3, has a radial velocity of  $-363$  km/sec,  $\mu_\alpha = -0.064$ ,  $\mu_\delta = 0.016$  and is near the solar apex. An interesting sidelight on HD 161817, approaching us at high speed, with small transverse motion, is that it will eventually become one of the brightest stars in the sky. If it has  $M = +2$ , its distance is now 100 parsecs, its  $V_r = 33$  km/sec; it will be only 9 parsecs from the sun in about 280,000 A.D., and of apparent magnitude  $+1.8$ !



bolic. The distribution perpendicular to the galactic plane, in the latter case, is also more concentrated than for the RR Lyrae stars.

If we used only luminosities derived from trigonometric parallaxes, 19 stars give  $M(\text{trig}) - M(H) = +0.6$  mag. (One object +38°4955, sdF6 (Popper) is omitted because its  $M = +10.3$  seems unreasonably large, although  $p = 0''.052 \pm 0''.013$ ). This difference is in the sense mentioned above, where two methods of obtaining relations between  $M$  and  $H$  differed by +0.8 mag. But the sense is such that, if only trigonometric parallaxes had been used, the sd and id stars would be +2.0 mag. below the main sequence. Our conclusions as to their luminosity are therefore conservative.

Both the spectral and luminosity analyses should be carried further. There is now good evidence that the sd and id stars can be distinguished from the ordinary wk

TABLE IV  
LUMINOSITIES OF SUBDWARFS

Type	No.	$M_{pg}$ from H sd + id	Main Sequence	$\Delta M_{pg}$ and m.e.
$\leq A5$	5	+3.5	+1.5	+2.0 $\pm$ 1.5
A6-F0	6	+5.2	+2.4	+2.8 $\pm$ 0.9
F1-F5	13	+5.0	+3.6	+1.4 $\pm$ 0.4
F6-F9	10	+6.3	+4.4	+1.9 $\pm$ 0.3
G0-G8	9	+7.4	+5.4	+2.0 $\pm$ 0.4

high-velocity stars, both in luminosity and in weakening of the lines. We do not know, however, whether there is a continuous band of stars below the main sequence from 0 to +3 mag. fainter, or whether there are discrete sequences. The wk stars need a similar but more elaborate analysis, and probably deviate less; stars like 10 CVn,  $\tau$  Cet and  $\mu$  Cas belong to the wk group, and are slightly below the main sequence.

4.3. *The velocity program.* The Palomar subdwarf spectra yield remarkably accurate velocities. One plate containing 30 measurable lines gives an internal mean error of  $\pm 0.6$  to  $\pm 1.0$  km/sec. Most subdwarf velocities, hitherto, had accuracies near  $\pm 5$  to  $\pm 10$  km/sec. Consequently, spectroscopic binaries can be found from very few observations. Since the main emphasis is on the spectral characteristics of these stars, two or more spectra will be obtained only for 40 of the 80 objects, repeating especially those whose velocities differed from published values [18].

Till now the only probable spectroscopic binary is  $-3^\circ 2525$ , (20C501), idF6, a  $9^m.5$  star. The published velocity of +25 km/sec. included several discordant measures; my two plates give  $+53.5 \pm 0.7$  and  $+41.0 \pm 0.6$  km/sec. As yet I have no idea of the period. The luminosity is about  $+6^m.5$ . Photoelectric observations for possible eclipse would be very important, because a direct determination of radius and mass has never been made for an extreme subdwarf. A few other objects have velocities differing by 10 km/sec from published results, but the majority have differences nearer 5 km/sec. The number of velocity variables found will be small, and the percentage of binaries must also be small. One general fact is that all the A5-G8 subdwarfs have sharp lines, that is, small rotation;  $-3^\circ 2525$  was first noted as suspect because of a slight rotational broadening.

4.4. *Composition.* Till now, the abundance determinations in high-velocity dwarfs are: three wk stars [20], where CH was found strong, the metals weak by small factors; two subdwarfs [21] in which a large reduction of the metallic abundances was indicated. L. H. Aller and the author are reanalyzing HD 19445, 140283, sdF5, and have added HD 161817, idA3 and HD 219617, sdG1; the author has investigated HD 103095, sdG8. The current interpretation of the high-velocity wk group is that it has perhaps one-half the population I metal abundance; the subdwarfs, sd and id, may have as little as one-twentieth the normal metal [21]. There is no evidence on the H/He ratio. With the low luminosities now found, the F and G subdwarfs need not have appreciably changed their initial H/He ratio by nuclear processes.

The kinematical properties run with the division into wk, id and sd stars. It may be conjectured that the sd are the purest and oldest type II population, and have the lowest metal abundances; the id are quite similar if less extreme, and the wk stars are ordinary "high-velocity" stars, that is, possibly a mixture of disk population II and old population I, with nearly normal abundances. The first recent attempt to give a model for the internal structure of homogeneous stars with small heavy-element abundance has been made by Reiz [22]; opacity arises from free-free transitions of H and He and energy from the proton-proton chain. In spite of many theoretical uncertainties, it is interesting to note that his H-R diagram for objects near a solar mass ([22], figure 1), homologous stars built on his model, lies +1.0 to +1.5 mag. below the main sequence and even further below for low hydrogen abundance.

If the subdwarfs are in fact shown to be metal-poor by detailed spectroscopic analysis, there are at least two possible evolutionary modes. Since kinematically they are extreme population II stars, they may have been formed from gases, near the galactic center, in the early days of our galaxy, before heavy elements had appreciably evolved in supernovae, novae or white dwarfs [23]. The low metal content favors Hoyle's views, rather than those of Gamow and collaborators; on Gamow's view the heavy element content and interstellar gas of the stars has not essentially changed. However, the possibility remains that star formation in the galactic center region was from an initially metal-poor gas, while the population I stars are formed from gas and dust. The dust is substantially enriched in metals and in C, N, O, with respect to H and to He.

The subdwarf problem is obviously related to the location of the main sequence of globular clusters. The giants in clusters show apparently low metal abundances, from spectra and from theoretical models. Baum states that in M 13 the so-called main sequence lies two magnitudes below the normal position [24].



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