PHILOSOPHICAL PROBLEMS OF THE STATISTICAL INTERPRETATION OF QUANTUM MECHANICS

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1. Classical physics

A striking characteristic of contemporary physics is the extensive utilization of statistical concepts. Statistical method is employed in the reduction of observations and in formulations of fundamental theory. It shall be the restricted purpose of this paper to discuss the role of statistics in basic physical theory.

The employment of statistical concepts in the construction of physical theory arises from the circumstance that observable physical phenomena are the resultants of large numbers of elementary processes. The reduction of large scale, macrophysical phenomena to fine scale, microphysical processes already required the use of statistical methods in classical physics. As background for the more recent quantum theoretical discussions I shall first sketch the function of statistics in classical physical theory. Classical physics was based upon the conception that observable physical phenomena have position within frames of space and time and proceed in conformity to causal laws. A causal law expresses regularity in phenomena, so that if specific data are initially given, possible data at other times can be inferred. Successful application of this conception is exemplified by the classical theories of mechanics, electromagnetism and thermodynamics. The development of fundamental theory for elementary, microphysical processes was initially molded by the foregoing classical conception. A classical microphysical theory assigned classically conceived physical quantities to elementary, microphysical processes. For example, in the kinetic theory of gases, which has served to explain large scale properties of gases, coordinates of position and components of velocity were attributed to the molecules of which the gas was assumed to be constituted. Now, the detailed specification of the state of a collection of molecules is impossible in practice. Simultaneous perception of all the molecules of a gas and therefore measurements of their simultaneous positions and velocities is beyond the power of human observation. Since an initial state of a collection of molecules could not be specified, states at other times could not be inferred with the aid of the causal laws of mechanics, which for classical microphysical theory had been extrapolated from the realm of large scale, macrophysical phenomena to the realm of fine scale, microphysical processes.

In the face of practical inability to specify in detail the microphysical state of a gas, the physicist had recourse to the theory of probability. It is well known that the definition of probability is subject to debate, but in physical theory one may proceed on the assumption that probability applies to events and is expressible as relative frequency within a collective. The application of probabilistic methods to the dynamics of a set of molecules, however, may be founded on some general theoretical considerations. A dynamical system in classical dynamics is described in terms of coordinates of position and components of momentum. The instantaneous state, or phase, of a system of molecules may be represented by a point in a multidimensional space in which the Cartesian coordinates of the representative point are the positions and momenta of the molecules. The changes of a system during time will be represented by progress along a line which is described by the representative point. The lack of knowledge of the microphysical state of a system prompts the consideration of an ensemble of similar systems the momentary states of which are represented by an aggregate of points distributed throughout a portion of the phase space. Continuously distributed points will move like an incompressible fluid, so that a specific aggregate of points will always occupy the same quantity of volume of phase space. This theorem of Liouville follows from the Hamiltonian form of the dynamical equations of motion and furnishes the basis for the hypothesis that equal elements of volume of phase space represent states of equal *a priori* probability. The definition of probability of a state therefore can be expressed as the fraction of volume of accessible phase space by which the specified state is represented. If the portion of phase space accessible to the representative point of a system is filled with points so that the density is uniform, then the probability of a state represented by a specific element of volume is expressible as the ratio of number of points in the element of volume to the total number of points. The probability of a state is thus expressible as the ratio of the number of systems in a specific element of volume to the total number in an ensemble. Thus we obtain probability as relative frequency with which systems in an ensemble in phase space are found in a characteristic element of volume. In classical statistical mechanics ensembles of systems have been employed to study the average properties of an individual system consisting of a large number of constituents. With the aid of the ergodic hypothesis, which recently has been the subject of contributions by Birkhoff and others, the average value of a function of the phase of an individual system was determined by calculating the average of the function over the ensemble distributed in phase space [1].

2. Quantum mechanics

We have seen that the employment of ensembles of systems by statistical mechanics characterized the reduction of macrophysical phenomena to microphysical processes in classical physics. I have now to explain that quantum mechanics has introduced statistical concepts of higher order. These concepts arise through limitation of applicability of classical concepts to elementary microphysical processes. The introductory account stated that classical concepts are suited to describe macrophysical phenomena. Electrons, photons and other microphysical elements of physical reality are only indirectly observable, but their characterization must utilize classical concepts which are used to interpret the experiments in which these microphysical entities produce observable results. The employment of classical concepts for the interpretation of experiments in which quantities belonging to elementary processes are measured, is limited by circumstances which Bohr has expressed by the concept of complementarity [2]. The most general empirical basis for this limitation of classical concepts is a dualism between corpuscular and undulatory properties of physical reality.

In the history of physics two points of view have been rivals for the interpretation of physical phenomena. Phenomena have been interpreted as manifestations of properties which have simple location, to use a term introduced by Whitehead, or as manifestations of properties of a field extended in space. In order to apply the point of view of simple location, physical phenomena have been explained as the action of corpuscles, or particles, which for mathematical theory may be idealized as physical points. The Newtonian theory of gravitational action at a distance between material particles exemplifies the idea of simple location. The point of view of the extended field is exemplified by classical electromagnetic theory which explains the transmission of electrical and magnetic actions by waves which are propagated through space. Since simple location connotes no extension and field connotes extension, the spatial properties connoted by simple location and by field are logically incompatible. Thus the concepts of corpuscle and wave as idealized for theoretical purposes demand applications that are mutually exclusive.

The dual aspects of physical reality which have required resolution of apparent contradictions by quantum mechanics may be set forth by the example of light. In experiments on interference and diffraction light exhibits properties which are readily explained in terms of waves; in experiments involving exchange of momentum and energy light manifests properties of corpuscles. There is no logical contradiction, however, because wave and corpuscular properties are not manifested simultaneously. An experimental arrangement whereby the wave properties of light are determined, its wave length for example, excludes the experimental determination of corpuscular properties, for example, position of a photon which is a corpuscular property. Wave and corpuscular properties are mutually exclusive but complementary. The application of the point of view of complementarity to dynamical concepts yields Heisenberg's principle of indeterminacy. As formulated for measures of a coordinate of position and its conjugate component of momentum, the principle states that the product of their standard deviations is equal to or greater than Planck's quantum of action divided by 4π [3].

In order to understand the situation which challenges our understanding it is desirable to distinguish between a physical quantity as measurable attribute of the physical world and the result of a measurement which is expressed by a number relative to some unit. In classical theory a physical attribute is presupposed to be possessed by a physical object independently of the physical context in which it manifests itself. In thought the physical attribute and the numerical value are thus interchangeable. In quantum theory the physical attribute is relative to a context of observation; the result of a measurement in general is not certain. Accordingly it is not appropriate to assume that out of its context of measurement the system possesses a definite attribute. Dirac has introduced the word observable as a substitute for the classical term physical quantity. In place of the distinction between a physical quantity as attribute and its values we have one between an observable and its values. The loosened connection between a system and its attributes is symbolized by the fact that the mathematical structure which stands for the observable of a system is the operator. The results of measurement of an observable are the characteristic values of the corresponding operator. The values of an observable in general are dispersed about a mean value. Thus we arrive at the higher type of statistical concepts which occur in quantum theory and to which I have already referred. The results of measurement, which could be idealized as unique in classical theory, now constitute a collective. Thus the properties of a quantum mechanical system are appropriately represented by an ensemble of systems.

For a statement of this result we need to introduce the central concept of the present discussion, that of the state of a system. We presuppose that methods are known whereby a system may be prepared so that it is in a determinate state, a state which has been called a pure case by Weyl [4], [5]. The pure case is represented by an ensemble which von Neumann has called unitary and in which all systems are in the same state [6]. In order to investigate an individual system in a determinate state we substitute a unitary ensemble of systems. The unitary ensemble may be described as one in which a specific set of observables will be found on maximum observation to yield certain values. In general, however, the results of measurement of an observable can not be predicted with certainty; the values of an observable are dispersed about a mean value. In addition to the unitary ensemble we may also have a mixture which is composed of a number of unitary ensembles in each of which measurement of specific observables will yield certain values respectively. The mixture substitutes for the system for which knowledge of its state is only partial. A unitary ensemble which represents a pure case is symbolized by a characteristic function which is frequently called a wave function, because in some interesting cases the function is the solution of a wave equation. The ontological status of the wave function is the central problem of this paper. The issue is: Is the state function objective or subjective? Does the characteristic function which represents a state represent physical reality or does it merely express a state of knowledge? An answer to this question requires the analysis of measurements to which I now turn.

3. Theory of measurement

Philosophical analysis of physical concepts during recent decades has led to agreement that the meaning of a physical concept is to be ascertained by discernment of the operations by which the concept is applied in the interpretation of phenomena. The nature of a physical quantity is manifested in the operations by which a value is found by measurement. Now measurement employs apparatus which exemplifies classical concepts and hence analysis of measurement must begin with classical physics. I take as the primary metrical quantity the distance between two points or the length of a line. The original operation of measuring length is based upon the establishment of superposition. We decide that the length of a certain body is a standard and then determine identity or nonidentity of a given length with the standard by superposing the latter upon the former. The numerical measure of length of a line is the number of times the standard can be laid off on the line. The measurement of time also involves the superposition of duration of a clock upon that of a given process. In order to define position in space we choose a frame of reference and choose a unit which is defined with respect to a standard. The position of a point is specified in terms of distances measured from axes of the frame. Position in time is similarly referred to some chosen origin. The measurement of spatial and temporal quantities occupies a special position in a theory of physical concepts, because the employment of superposition in the operation of measurement is founded upon the identity of property measured and standard. The establishment of superposition for the cognition of identity is presupposed not to involve interaction between object of measurement and instrument. The perception of superposition is a macrophysical process and its idealization for theoretical purposes abstracts from gravitational or other action between the bodies that are superposed.

Length and duration are in a sense intuitively exhibited properties. Physical properties, however, are otherwise dispositional attributes which manifest themselves only in interaction. For example, momentum is ascribed to a macrophysical body on the basis of its changes in motion during interaction with other bodies. The principle of conservation of momentum serves as a definition for the measurement of momentum. Similarly we ascribe electrical charge to a body on the basis of interactions between charged bodies. Elementary bodies such as electrons are not perceptible in the usual sense and one is completely dependent upon interactions with apparatus in order to measure their properties. In the operation of measuring a dispositional attribute the space time processes of a measuring instrument are observed and classical concepts are employed to determine the physical quantity which is ascribed to the object of measurement. Thus in the measurement of properties of microphysical objects there occurs an interaction with some apparatus which reacts upon the object acting upon it. For future use I distinguish between the process of registration, in which the object registers some observable effect on the apparatus, and the subsequent process of perception in the psychological sense, in which some human observer perceives what has been registered.

The interaction between object and apparatus requires that in principle one create a partition between them. The physical state of the apparatus on the observer's side of the partition is beyond consideration while the apparatus is serving its function for the investigation of the object on the other side. In classical physics the function of the partition can be ignored for the reason that the action of the apparatus on the object can be made vanishingly small in principle. According to quantum theory there is a finite lower limit to the action of apparatus upon the object during measurement of a microphysical element of physical reality. The consequence is a limitation in the application of classical concepts for the description of microphysical reality. This can be explained by an example of Bohr in which a particle passes through a slit in a diaphragm [7]. Preparation for measurement of position requires that the diaphragm be rigidly fixed to a support which defines the space frame of reference. During its passage through the slit the particle exchanges momentum with the diaphragm; the momentum acquired by the diaphragm is absorbed by the supporting space frame of reference and therefore cannot be used to calculate the momentum acquired by the particle. On the other hand, preparation for measurement of momentum requires that the diaphragm be left mobile. It is possible in principle to measure its momentum before and after the passage of the particle, and thus to calculate the momentum of the particle after it has passed through the slit. But determination of momentum of the diaphragm requires a collision with a test body during which process there is an uncontrollable displacement of the diaphragm. Thus we lose knowledge of the particle's position when it passed through the slit. Bohr has stated that one must discriminate between essentially different experimental arrangements and procedures, which are suited either for an unambiguous use of the idea of space location, or for a legitimate application of the principle of conservation of momentum. With each of the mutually exclusive experimental arrangements we are confronted, not merely with ignorance of the values of specific physical quantities, but with the impossibility of defining the concepts of these quantities in an unambiguous way.

Thus the partition between object and apparatus imposes a restriction upon the scope of our physical investigation. We may, however, look upon the situation from a higher point of view. We may, for example, select a frame with respect to which the interaction between electron and diaphragm is an object to be described by the methods of quantum mechanics. Upon using the new frame to determine spatial positions for electron and diaphragm, we must renounce control over its momentum. The position of the partition between object of observation and measuring apparatus of the observer can be changed arbitrarily with a corresponding change in the object of study. But definition of the object of any investigation requires that such a partition be set up. Heisenberg has set forth the view that the partition, or "cut," is the seat of the indeterminacy introduced by quantum mechanics [8]. On the object side of the cut states of physical systems are transformed in conformity to deterministic laws. On the observer's side of the cut processes travel through the apparatus in conformity to classical laws and register a perceptible result. The uncontrollable disturbance of the object by the apparatus provides the free play at the cut which is necessary to provide compatibility of the predictions of quantum mechanics with the indications of classical measuring apparatus.

4. Relativity of state

We have seen that the state function represents a state of affairs on the object side of the partition. It is further necessary to recognize that a representation of state is relative to the context of observation. The state function has a relativity which replaces the absoluteness expressed by classical concepts in the sense previously explained. The need to abandon the absoluteness of classical physical description in quantum theory may be shown by examples given by Bohr [7] and C. F. von Weizsäcker. We have already considered Bohr's example in which a particle passes through a diaphragm and in which position or momentum may be measured depending upon whether the diaphragm is rigidly fixed to the space frame of reference or is left mobile. It is not the physical interaction between object and instrument as such which fixes the quantity which can be measured, for the arrangement for cognition plays a role. In the example of the particle which passes through a mobile diaphragm, after this passage we are still left with a free choice as to whether we wish to know the momentum of the particle or its initial position relative to the rest of the apparatus. We may either measure the momentum of the diaphragm and use it to calculate that of the particle, or we may attach the diaphragm to the support, thereby fixing position but renouncing the possibility of determining momentum.

C. F. von Weizsäcker has given a further example of this point [9]. Suppose that an electron is known to be in a given plane; its position within the plane can be found by illuminating the electron. We further suppose that the light used is of low intensity, so that a single photon is scattered. In order to determine position one would place a photographic plate in the appropriate image plane. The light scattered by the illuminated electron would then be brought to a focus on the photographic plate; from the position of the image and the laws of classical optics one could find the position of the electron at the time of collision with the photon. In this application of the wave picture to light one uses the representation of a spherical wave which passes through the entire lens of the microscope. Thus no definite direction characterizes the motion of the photon from its place of collision with the electron and so the change of momentum which occurs to the electron cannot be determined precisely. After the collision the electron will be characterized by a wave function which specifies a sharp position, but a less sharp momentum than in the previous state. One creates a new mode of description of the collision of electron and photon if one places the photographic plate in the focal plane of the microscope. All light rays which approach the microscope from the same direction will then be united in the focal plane. The photographic plate will show a sharp image, for the photon has only enough energy to blacken the plate at one point. The point in the focal plane where the photon strikes the plate is characteristic of a definite direction from which the light approaches the microscope. The direction is definite, but the place in the object plane from which the light started cannot be determined. If the momentum of the photon were known before the collision, one could determine its change of momentum from its direction after collision and hence calculate the change in momentum of the electron. The electron would then be represented by a state function which specifies sharp momentum and correspondingly unsharp position. In both experimental arrangements of von Weizsäcker's example the same event, collision with a photon, occurs to the electron, but different wave functions are assigned to it according to the mode of observation. After the photon collides with the electron and passes through the lens, one could in principle place the photographic plate in the image or focal plane, and thereby determine respectively position or momentum of the electron. The intellectual act of preparing a well defined arrangement for observation is essential for the operation of measurement.

The foregoing example illustrates the fact that quantum mechanical description does not pertain to the physical system in itself as does a classical description, independently of the observations through which one has taken cognizance of it. In quantum mechanics no sharp separation can be made between an independent behavior of objects and their interaction with measuring apparatus. The state of a system as represented by a state function does not satisfy the criterion that reality is independent of the mode of cognizing it.

5. Measurement of quantized systems

As preparation for utilization by quantum mechanics of ensembles of systems to represent the states of microphysical systems, it will be helpful to consider the behavior of quantized, microphysical systems upon measurement. Let us then have given a system which has been prepared so that it is in a pure, or determinate, state. This means that characteristic values of operators which correspond to a maximum number of observables can be predicted with certainty. But the results of measuring other observables which pertain to the system are uncertain. In each of these cases any one of a spectrum of characteristic values may be found. The system will be thrown into one of a number of characteristic states, each of which is characterized by values found for specific observables. The preparation for measurement makes it appropriate to represent the prepared determinate state as a superposition of component states. The mathematical correlate is the expansion of the given state function in terms of the characteristic states of the observable to be measured. The squares of the absolute values of the coefficients of the expansion are the probabilities of finding the system in the corresponding states upon observation.

The usual example for the behavior of quantized systems on measurement is polarization. A beam of light passes through a Nicol prism and the light which emerges is plane polarized. If one conceives of light as consisting of photons, the plane polarized light will then consist of photons polarized in a given direction. Suppose that a single photon passes through a second Nicol; the state function for the photon is to be expressed as a superposition of two functions, each of which characterizes a photon polarized in one of two mutually perpendicular directions. But the measurement has not yet been completed, for the two component state functions are still able to interfere with each other. Let us suppose that one component is allowed to fall upon a photographic plate. The photon will be absorbed as it hits the plate, so that any further interference between the component state functions is now excluded. The action of the photon on the photographic plate is registration of an effect of the object upon the measuring apparatus. This example illustrates the type of preparation which is required in a procedure of measurement. If one wishes to measure momentum, one may allow a photon to pass through a diffraction grating which we take account of by expanding a state function into a set of orthogonal state functions. An additional factor then causes a discontinuous change in the observed object to a state represented by one of the component state functions.

The grating as example of physical apparatus illustrates that for quantum theory an instrument of measurement serves as a sieve which resolves a state into component states. The aggregate of states which may result from measurement is represented by directions in an abstract Hilbert space of infinitely many dimensions. In this linear vector space the state function may be viewed as a unit vector, or point. This space is spanned by orthogonal characteristic functions as base vectors which define axes for the space. The setting up of a grating is preparation for measurement and is represented by the expansion of the state function according to the base vectors. The procedure of measurement throws the system from a given state into a component state. This process is represented by the operation of normal projection of the given state vector upon an axis, or more generally a subspace. The square of the absolute value of the normal projection of a unit state vector upon a subspace is the probability of occurrence of the resulting state.

To describe the results of measurement we have recourse to representation of states by ensembles of systems. Initially the system as a pure case is represented by a unitary ensemble. Upon registration in measurement the system becomes a mixture. Perception of the registration constitutes selection out of the mixture of those systems for which the observable measured will certainly be found to have a specific value.

I have remarked that after passage through a Nicol an additional factor is needed to cause the discontinuous change of the system to a state represented by one of the component states. And here we come to a debatable issue. Is the factor that produces the decision an objective one or subjective? P. Jordan [10] interprets von Neumann [6] to hold that the conversion from pure case into mixture is a mental process of the observer. If the observer forgets those relations between the component wave functions which make them able to interfere with each other, then in the mind of this observer the pure case is turned into a mixture. According to this interpretation of the subjective view the state function expresses the actual knowledge of the observer and not his potential knowledge. P. Jordan [10] and H. Margenau [11] adopt an objective view: the decision between the various possibilities is made by a physical process such as the absorption of a photon by a photographic plate. The decision is made by the registration which is a macrophysical factor in the observation and is independent of perception. It appears to me that the physical interpretation of the process of decision is also that of Dirac [12].

6. Resolution of paradox

Against the background now provided we shall be able to discuss the clarification by statistical considerations of certain paradoxes which have been expounded by Einstein in collaboration with Podolsky and Rosen [13]. They have argued for the thesis that quantum mechanical description of physical reality is incomplete. The discussion is based upon the following criterion of physical reality: "If, without in any way disturbing a system, we can predict with certainty (that is, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." The argument considers an example in which two systems interact for a while, and then cease to interact. It is possible to measure either the position or momentum of one system and infer the position or momentum respectively of the other. Since a measurement on the first system occurs after it becomes independent of the second, Einstein, Podolsky and Rosen concluded that position and momentum should be ascribed physical reality simultaneously. Quantum mechanics was interpreted to be incomplete since it does not permit the simultaneous prediction of results of measurements of these two quantities with certainty. The resolution of the apparent paradox is provided by the complete statistical formulation of the theory.

For this discussion we need a more explicit analysis of the procedure of measurement than has been given thus far. For the following analysis I am indebted to a discussion by A. Kratzer [14]. Let us then have given that a system is in a state which is represented by the function φ. It is proposed to measure an observable L for which one of a set of characteristic values may be found. The spectrum of possibilities for measurement is represented by expanding the state function φ in terms of characteristic functions χ_i of the observable, $\varphi = \sum a_i \chi_i$. On performing a measurement on observable L, there occurs a transition from φ to χ_i, where χ_i results with relative frequency $|a_i|^2$. Cognition of a particular characteristic value of the observable shows that there has been a transition to the associated characteristic state. The function φ which represents the system in a determinate state, is to be interpreted as representing an ensemble of systems in which a characteristic value and its corresponding characteristic state can be found on measurement with a frequency which is the square of the absolute value of the coefficient in the expansion. The process of measurement involves two factors: (1) Interaction between object and measuring apparatus which causes a discontinuous transition with a calculable probability to a new state; (2) Cognition of the particular state which is produced.

Now the interaction between object and measuring apparatus can be made subject to theory. Accordingly we displace the partition between object and observer, so that the object of study includes the apparatus which was initially part of the observer in the broadest sense of the term. This presupposes that enough of the measuring apparatus remains on the observer's side of the partition so as to make possible measurements on the new and expanded object. Let us suppose that the original object and apparatus are given initially as independent systems, the first large, the second small, so that their states are respectively described by

$$X = \sum a_i \chi_i$$

and

$$\Phi = \sum b_i \Phi_i \, .$$

At an initial time the two systems are in states represented by $\chi_n(x_1)$ and $\Phi_m(x_2)$. The systems now enter into interaction during which time the total system is represented by $\Psi(x_1, x_2)$ which is expanded as

$$\Psi = \sum a_{ik} \chi_i \Phi_k \, .$$

In the total system the precisely defined initial states of the partial systems have been lost.

Let an observable F be measured on the large system and the state Φ_j be cognized. The state of the total system then becomes

$$\Psi' = \sum_i a_{ij} \chi_i \Phi_j \, .$$

The probability that an observable L for the small system will be found to have the value l_i is $|a_{ij}|^2$.

Suppose that instead of measuring F we had measured on the large system quantity G which does not commute with F. The characteristic functions of G are Γ_i which are related to Φ_k as follows,

$$\Phi_k = \sum_i s_{ik}\Gamma_i .$$

We now represent the state of the total system by an expansion according to $\chi_i\Gamma_m$.

$$\Psi = \sum b_{im}\chi_i\Gamma_m ,$$

where

$$b_{im} = \sum_k a_{ik}s_{mk} .$$

If there is a measurement of G and the presence of Γ_h is determined, the state of the total system becomes

$$\Psi'' = \sum b_{ih}\chi_i\Gamma_h ,$$

and the probability of finding l_i on the small system is $|b_{ih}|^2$, a result which differs from $|a_{ij}|^2$. We thus have the paradoxical result that the expectation value of a physical quantity of a system depends on which quantity is measured on another system which is no longer coupled with it.

The resolution of the apparent paradox is as follows: Prior to measurements on the partial systems, they were coupled so that the properties of the systems were related. After the coupling has ceased, an extension of our knowledge of the large system carries with it an extension of our knowledge of the small system. Another kind of cognition about the large system furnishes a new kind of cognition about the small system. The paradoxical result that the expectation value of a quantity on the small system depends on what happens to the large system at a time when the two systems are no longer coupled is eliminated by noting that our formulae always refer to an ensemble of systems and make assertions about its statistics.

The measurement of F on the large system produces an arrangement in the states $\chi_i\Phi_k$, and cognition of Φ_j is represented by selection of states $\chi_1\Phi_j$, $\chi_2\Phi_j$, . . . , $\chi_i\Phi_j$ out of the ensemble Ψ'. The total system after cognition of the result of a measurement of F is represented by an ensemble of systems in which the systems are in states $\chi_1\Phi_j$, $\chi_2\Phi_j$, . . . , and this new ensemble represents the two systems in relation to one another. If one measures G of the large system, one arranges the states $\chi_i\Gamma_m$; cognition of G_h signifies a selection of systems in states $\chi_1\Gamma_h$, $\chi_2\Gamma_h$, . . . , $\chi_i\Gamma_h$ out of the totality Ψ''. Thus we construct a new ensemble of systems to represent the relation of the two systems, an ensemble which is selected by a measurement of G on the large system. The two ensembles, one selected with respect to F and the other with respect to G, will yield different expectation values for observable L of the small system. This is not the consequence of influence on the small system by measurements on the large, but the result of a different selection for the ensembles.

The preceding analysis enables one to explain the previously cited example of Einstein, Podolsky and Rosen. The mathematical formulation fits an experiment in which two particles interact while each passes through a separate slit in a diaphragm. If after passage the position of one particle is fixed, the position of the other can be inferred from the distance between the slits. If the momentum of one particle is determined, the momentum of the other can be inferred from the total momentum of the particles. When interaction of the two particles ceases we can infer either position or momentum of the second from position or momentum respectively of the first. When position of one particle is determined, the ensemble which represents the system resulting from the measurement is one in which the first particle has a fixed position in all systems. The coupling of the two particles was such that the position of the second could be predicted with certainty. However, knowledge of momentum is completely lost; the systems may be found in any one of the possible states of momentum. On the other hand, if momentum of one particle is determined the ensemble comprises systems in which the given particle has a determinate momentum. The nature of the coupling during passage of the particles through the slits was such that the momentum of the other particle can be determined. But knowledge of position is completely lost; the systems may be found in any one of the possible positions. The possibility of predicting either position or momentum of a particle by a corresponding measurement on an independent particle previously coupled with it, does not mean that an independent system influences another. The result of one type of measurement is a system which is represented by an ensemble that is selected in one way out of the original ensemble representing two systems which have been coupled; the result of another type of measurement is represented by an ensemble selected in a different manner. Each type of ensemble which is selected by a measurement permits its own type of prediction for measurements of some other observable.

7. Philosophical conclusions

I now draw some philosophical conclusions from the preceding analysis.

The basic issue has been formulated: Is the state function subjective or objective? A preliminary answer is that it possesses both aspects. From the very nature of science the symbol for a state function must designate a conceptual structure which is a constituent of our knowledge of physical reality. The history of ideas shows that a concept introduced for theoretical purposes comes to express an attribute of reality. Thus the dichotomy between the state function as expressing a state of knowledge and as representing an objective state of affairs is not proper. Any physical concept which is thoroughly certified for cognition of reality must be considered objective. That the state function may be complex does not lessen its cognitive capacity. As medium of a state of knowledge the state function expresses some property of an objective system.

The issue is more properly to be drawn between absoluteness and relativity of physical properties. Classical physics presupposed absoluteness of physical quantities in the sense that measurable properties such as length, mass and electric charge were considered to be independent of the context of observation. The special theory of relativity introduced relativity of spatial and temporal quantities for physical description. However, all observers employing the same space time frame ascribe the same geometrical and temporal properties to a given process. Quantum theory has now introduced an additional relativity, one to context of observation for an individual observer. The properties ascribed to an element of physical reality depend upon the type of experiment employed by the observer to investigate it. We must then recognize that the state function represents an objective state of affairs which is relative to the experimental arrangement.

While the state function represents an objective situation which is relative to an experimental arrangement, the objective state of affairs is a very special one for quantum mechanics. A quantized, microphysical system is a system of possibilities for observation. The attributes in general are not interpreted as having reality independent of observation as in classical physics. For independent reality in this case we need only the criterion of physical reality which has been quoted from Einstein, Podolsky and Rosen. Accordingly, the objective situation for quantum mechanics is appropriately represented by a virtual ensemble of systems. Experiments are performed with apparatus which embodies the laws of classical physics. The concepts for the interpretation of physical experiments on microphysical objects are limited in their use. But they are all we have to characterize the properties of microphysical systems from interactions with macrophysical apparatus. The limitations of concepts of quantities which are found by mutually exclusive methods expresses itself by the replacement of classical deterministic relations between results of measurement by statistical ones. The symbol of this circumstance is the representation of a single system in a determinate state by a virtual, that is, fictitious, ensemble of systems. Such a virtual ensemble of systems represents the manifold of possibilities of measurement offered by a quantum mechanical system.

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