Preface

The 40-th Finsler Symposium on Finsler Geometry was held in the period 2005.09.06 – 2005.09.10 at Hokkaido Tokai University, Sapporo, Japan. The Symposium's purpose was not only the usual annual meeting of the Finsler geometers from Japan and abroad, but also to commemorate the memory of the late Professor Makoto Matsumoto.

This volume contains surveys and original articles based on the talks given at the Symposium by some of the participants as well as contributions from authors that for different reasons were not able to attend the Symposium in Japan, but agreed to contribute to this memorial volume. The editors would like to express here their gratitude to everybody who contributed in one way or another to the publishing of this volume. Especially, we thank to D. Bao for many advices and useful suggestions.

The authors were invited based on their past joint research with Professor Matsumoto, or for researching important subjects in Finsler geometry influenced by Professor Matsumoto's work.

We have organized the scientific papers into three parts from the point of view of their contents. Namely, Part II concerns topics on Finsler geometry more or less directly related to the work of Professor Matsumoto. We would like to emphasize here especially one of the most famous open problems in Finsler Geometry: to determine a concrete example of a Finsler metric that is Landsberg but not Berwald. The search for this kind of metric, which everybody wanted to see but no one could actually get, makes D. Bao to call these metrics Unicorns in Finsler Geometry. The first two papers in the present volume bring a new light to the Unicorn problem in Finsler Geometry. Part III encompasses articles concerning topics inspired by, or adjacent to, classical Finsler geometry as the Geometry of Paths, Dynamical Systems, or the Geometry of Sprays on Finsler manifolds. The last part contains two comprehensive surveys in Complex Finsler geometry. We consider that the papers containing fundamental topics of Finsler geometry are interesting not only for specialists in Finsler geometry, but for specialists in Riemannian geometry or other fields of differential geometry also.

The editors would like to express their gratitude to the ASPM Editorial Board and especially to Professor Y. Giga who suggested that we edit this volume.

We are going to give in the following a synopsis of the papers in this volume.

David BAO. The paper contains an excellent introduction of the notion of Finsler manifold. Some fundamental topics as geodesic sprays and the notion of parallel transport are discussed in detail.

Moreover, the paper is a very good initiation for specialists and non-specialists in Finsler geometry in two important research area in the field: the search for Finslerian "unicorns" (i.e. Landsberg metrics that are not Berwald ones) and the study of Ricci flow.

Sándor BÁCSÓ, Xinyue CHENG and Zhongmin SHEN. The paper gives a nice review of the main results in the geometry of Finsler spaces with (α, β) metrics. These are important examples of Finsler metrics viewed as deformations of Riemannian metrics. The highlight of the paper is §6 where the authors give some concrete almost regular examples of Finslerian unicorns in dimension greater than two.

Ioan Radu PETER. There are a number of comparison results for Riemannian manifolds of positive sectional curvature which can be regarded as connectedness results for an isometric immersion. The present paper is an attempt to generalize these classical results to the Finslerian setting.

Sorin V. SABAU and Hideo SHIMADA. This paper contains a review of main results on Finsler surfaces as well as an extension of the Gauss-Bonnet theorem to the case of a Landsberg surface with smooth boundary.

Lajos TAMÁSSY. The paper is a good survey of the geometry of a Finsler space regarded as a metric space. Basic notions in metric spaces like distance function, angles, volumes, etc. are discussed. In the third part, the theory of metrical connections is presented from this new perspective.

Peter L. ANTONELLI and Solange F. RUTZ. The authors study the projective geometry of sprays on Finsler manifolds, dealing especially with the problem of classifying sprays on Finsler surfaces whose coefficients are linear functions of position only. Applications to stochastic models with noise in biology are also discussed.

Mike CRAMPIN and David J. SAUNDERS. The article considers an important topic related to the geometry of Finsler spaces: the geometry of paths. After describing the main working frame for the path geometry, the authors show an interesting and completely new result.

Namely, every path geometry leads to an associated almost Grassmann structure.

Rezső L. LOVAS, Johanna PÉK and József SZILASI. The authors study the theory of connections on a (generalized) Finsler manifold. Namely, they consider the case of Finsler connections induced by a given Ehresmann connection on the total space of a vector bundle. Concrete examples of connections and metrics make the whole construction a tool ready to be used in applications.

Radu MIRON. The paper studies the differential geometry associated to a dynamical system defined on a Lagrange space taking into account the relation between the integral lines of the dynamical system and the paths given by the Euler-Lagrange equations on the Lagrange space. The originality of the present paper is the extension through Legendre transformation of this geometrical theory to one of the dynamical systems defined on the $\mathcal{L}$–dual Hamilton manifold. The particular case of Finsler–Cartan $\mathcal{L}$–duality shows the utility of this theory in applications.

Tadashi AIKOU. The author gives a good survey of recent developments in complex vector bundles geometry of Finsler type. Notions like ampleness, negative holomorphic vector bundles (Kobayashi's theorem) and the geometry of the Chern–Finsler associated connection are discussed. A special case of the whole theory is the case of Finsler-Kähler manifolds. The characterization of Finsler-Kähler manifolds is studied using the Cartan connection, which is naturally induced on the real tangent bundle from the Chern–Finsler connection.

Pit-Mann WONG. The paper covers a quite large thematic area starting with a very good introduction and continuing by describing the fundamentals of modern complex Finsler geometry. By discussing the interplay between differential geometry, algebraic geometry and complex analysis the author convinces readers that complex Finsler geometry is a very natural extension of all these fields. The paper is a very good reference in complex Finsler geometry, and for non-specialists in Finsler geometry also.

Sorin V. Sabau
Hideo Shimada

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.