Foreword

When Takayuki Hibi invited me to the conference on Gröbner bases with the subtitle "The 50th Anniversary of Gröbner Bases," I felt deeply pleased and grateful that the subject of my PhD thesis back in 1965 was receiving and is receiving so much attention.

In fact, the beginning in 1965 was quite hard and I had absolutely no feeling for what the impact of my work could be. In 1964, I was a working student of mathematics with a full-time job as a programmer at the computing center of the University of Innsbruck. I had thoroughly studied Van der Waerden's and Gröbner's book on commutative algebra (polynomial ideal theory) and I was determined (in spring 1964) to take the first problem Gröbner would present in his research seminar as my PhD thesis problem in order to finish my study as quickly as possible. By this, I hoped that my strenuous life as both a student and a programmer would soon be over. Luckily (in retrospect), the first problem Gröbner presented was the problem of constructing, by an algorithm, a linearly independent basis for the residue ring of a polynomial ring modulo a polynomial ideal.

Unfortunately (in my subjective feeling at that time), the problem turned out to be quite tough although its specification was so easy to understand. Thus, I still suffered through quite a long time of working, during the day, as a programmer and working, in my "free time" but often also while waiting for computer output, on Gröbner's problem. The problem was basically always on my mind and I alternated between looking to examples, trying to invent and prove some lemmata and experimenting with the computer. I will always remember the big relief I felt when one day — I was riding on my bike to the university — I suddenly "saw" that the clue to the solution of the problem would be the consideration of what I then called the "S-polynomials." From there it was a comparatively short step to come up with a proof of the S-polynomial theorem and, in parallel, with the S-polynomial algorithm for computing the special kind of ideal bases which I later called "Gröbner" bases. And, it was then clear that, as soon as one has a Gröbner basis for an ideal, the original problem of Gröbner and also a couple of other fundamental problems about polynomial ideals could be solved by algorithms.

In retrospect, what appeared for me to be a big tension between pure mathematics, theorems, structure, and proving on the one side and algorithms, data structures, computer hardware intricacies, programming languages and efficiency on the other hand, spanned the axes for my entire professional life.

My work did not get any attention in the first years after submitting my PhD thesis in the last days of 1965 (not even after appearance of my 1970 aequationes paper) but, starting from 1976, many people both in pure mathematics and applications areas became heavily interested. When I observed this, I started to understand that Gröbner, by giving me this tough problem, had in fact provided a big intellectual gift to me and I decided to attach his name to the type of bases I had invented in my thesis. While, in 1964, even after having studied the books of Van der Waerden and Gröbner, I did not really understand the importance of the problem (but apparently had the intellectual freshness to solve it) Gröbner surely had been fully aware of the crucial relevance of the problem. He had an enormously broad view and mastery of mathematics always keeping coherence between analysis, algebra and geometry.

Over the years, I tremendously enjoyed my life between the world of abstract mathematics and algorithms. I am grateful for every moment and I am deeply impressed by all the wonderful contributions so many people have made to the theory, algorithmics and applications of Gröbner bases. In the future, we will move to higher and higher levels of algorithmic mathematics, in particular by moving from the object level of mathematics to the meta-level.

By Gödel, there is no upper bound to iterating these moves to higher levels and it is very important to understand and make it clear that, whatever catch words (like "artificial intelligence," "machine learning," "deep reasoning," etc.) people outside mathematics like to invent for these moves, the intellectual skill and culture that keeps this upward movement going, is just the ever-young power of human mathematical thinking. Thus, we can look forward with enthusiasm and optimism to the next layers and waves of algorithmic mathematics with more and more challenges for the brightest minds of the next generations of mathematicians.

Let me conclude with warm thanks to Takayuki Hibi for organizing the conference and guiding us in the process of preparing our papers for the proceedings.

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