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EVIDENCE OF A 6.12×10^{18} GeV PARTICLE: DETECTION AND MATHEMATICS

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Abstract. In a new approach the graviton is defined as the field particle of spacetime rather than the mediator of gravity. The unification equation is derived and used to predict that for a freely falling body, the energy of incident gravitons is 6.12×10^{18} GeV. Redshift and scattering of gravitons should produce diffraction patterns, galactic halos and expansion of the Universe. The energy of incident gravitons remains constant as the Universe evolves because of the Doppler shift as bodies fall towards redshifted gravitons. Complex space is used to represent gravitons and explain Young's two-slit interference. The approach is corroborated by empirical data and extends establish theory.

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CONTENTS

1. Introduction

The theories of relativity and quantum physics have provided science with tremendous depth and detail in the quest to understand the Universe, however these two pillars of modern physics are unable to explain several well-known observations. Difficulties are highlighted by the findings of pioneering cosmologists who revealed that the motion of stellar bodies cannot be explained by traditional physics alone [7, 12, 19]. Nowadays, in order to interpret the phenomena of galactic halos and accelerated expansion of the Universe, the current cosmological model includes the *ad hoc* inventions of dark matter and dark energy. The magnitude of the difficulties with the cosmological model are confirmed by recent studies with the Planck Instrument which enabled six key parameters to be determined with unprecedented precision, and confirm that, according to established theory, 95 % of the Universe is of unknown form [1]. In addition, physics has no mechanism for explaining quantum phenomena, such as Young's two-slit interference experiment [4].

Such difficulties have led some scientists to conclude that despite the plethora of new and precise data, general relativity and quantum theory are like two parallel tubes, whereby gravity does not care about particle physics [6]. Other scientists have concluded that both cosmology and particle theory need new physics [9]. Hence, the aim of this paper is to show that the current problems of modern physics can be solved with a new approach. The work presented in this paper is based on the proposition that there exists a quantum field particle which provides the existence of spacetime itself.

2. A Mechanism for Spacetime

Quantum theory and general relativity both describe, in their own way, the motion and actions of matter in a background of space and time (or spacetime), however there is one feature which is lacking from these established theories. Both lack a mechanism for producing spacetime. This gap can be filled by proposing that a quantum field particle, the graviton, is responsible for providing the Universe with space and time, and that curvature of spacetime is just one phenomenon that the particle provides [13, 14, 15, 16, 17]. This proposition is explained in the following way.

The graviton is generally referred to as the quantum field particle responsible for mediating the gravitational force. This concept is suspect because if the graviton behaves in a similar fashion to the photon, it would imply that every mass should continuously lose energy as it curves spacetime, and this loss of energy is not observed [5]. Also, even though all quantum particles are influenced by curved spacetime (i.e., gravity), the cosmological constant G is missing from the equations of quantum theory. As a result the graviton has become such an inscrutable part of quantum theory that many texts simply ignore it, or argue that its energy must be infinitesimal because gravity is the weakest of the four forces of nature. Hence, particle physicists and cosmologists generally consider the graviton to be an unsubstantiated minor part of their standard models [11].

This paper has a different point of view and redefines the graviton as the field particle providing the Universe with space and time; that is to say, the graviton is the spacetime-particle. Accordingly, the graviton has properties which enable it to link points in space and time. All mass exists in a sea of gravitons and mass is continuously encountering incident gravitons. In order to enable changes to take place, gravitons are capable of carrying photons and other quantum particles. So, if an excited particle emits a quantum of energy, such as a photon, a set of incident gravitons carries the emitted quantum in the time dimension. Thereafter, other gravitons can join the process of carrying the quantum of energy until it is detected (or absorbed) by another particle. Thereafter the newly excited particle may emit the quantum of energy and another set of gravitons shares in carrying the quantum, and the process is repeated. In effect, a set of gravitons shares in carrying a quantum of energy in the time dimension to another location in space. By this process a quantum of energy is lost by one particle at a certain point in spacetime and another particle gains the quantum of energy at another point in spacetime. This description implies that gravitons have a dual mechanism which provides the Universe with the dimensions of space and time, and enables changes to take place. This basic description will be developed in the paper.

Logically, each incident graviton imparts space and time in equal measure, so it follows from the ratio that all gravitons are at c. Hence the quantity commonly known as the speed of light c is actually a characteristic of the graviton. Photons, and other massless particles, are carried by sets of gravitons at c, and this quantity is independent of the motion of the particle which emits the photons. Massive particles are also carried by a set of gravitons, and the gravitons in the set are constantly being replaced because the velocity of massive particles is less than c. Accordingly, the sea of gravitons forms the spacetime continuum and no point has a more special reference frame than any other point. Thus the graviton is the mechanism which underpins the two postulates of special relativity [2] and the actions of gravitons can be restated in terms of two propositions as follows.

Proposition 1. *The dimensions of space and time are provided by the graviton, the field particle of spacetime, which links points at c and carries the energy of quantum particles in the time dimension.*

This proposition implies that the all points in the Universe are teeming with gravitons. The spacetime continuum (or cosmological fluid) began when gravitons were first formed and, since then, gravitons have provided spacetime on an ongoing basis. The energy of particles is carried by sets of gravitons that link points in spacetime.

Proposition 2. *Encounters between a set of incident gravitons and a mass produce a redshift and scattering of emitted gravitons. Because of these encounters, gravitons transfer energy to mass and this energy is the energy-content of mass.*

This proposition implies that the graviton is the mechanism which provides quantum particles with their energy-content (i.e., the energy-content that was first proposed by Einstein [3]). It also highlights the requirement to consider both redshift and scattering of gravitons when calculating curvature of spacetime.

This paper will show how these two propositions provide a foundation for the two pillars of modern physics. We will see that the path we are taking provides simplicity instead of complexity, and that it leads to straightforward explanations of the aforementioned problems of modern physics.

3. Deriving the Unification Equation

In this section propositions (1) and (2) are used to derive the equation of gravitational redshift of general relativity and the equation which unifies established theories [13, 14]. Consider the number N_{X0} of incident gravitons on a region in spacetime and let the incident gravitons have frequency f_{X0} and the region have radius r with surface area $4\pi r^2$. The region contains mass M and, according to proposition (2), the gravitons are redshifted and scattered by the mass. Hence the number N_{XE} of emitted gravitons is less than N_{X0} and the frequency f_{XE} of emitted gravitons is less than f_{X0}. The number of incident gravitons encountering a length l is related to the wavenumber k_{X0} of incident gravitons by: $k_{X0} = \dfrac{N_{X0}}{l}$ and similarly, the wavenumber k_{XE} of emitted gravitons is given by: $k_{XE} = \dfrac{N_{XE}}{l}\cdot$ The energy E_X lost by incident gravitons to M is given by

$$E_X = (hf_{X0}k_{X0}l - hf_{XE}k_{XE}l), \qquad \frac{E_X}{l} = (hf_{X0}k_{X0} - hf_{XE}k_{XE}). \tag{1}$$

The energy-content E_M of M is within the surface and is given by: $\dfrac{E_M}{4\pi r^2}$, which can be equated to the energy transferred from the gravitons, as follows

$$\frac{d(\frac{E_X}{l})}{\mathrm{d}r} = \frac{E_M}{4\pi r^2}. \tag{2}$$

The change in energy of gravitons is obtained by treating this relation as a separable equation and integrating r from radius R to ∞. This change becomes infinitesimal for an infinitely large value of R and as a result, the transfer of energy from gravitons to M is given by

$$(hf_{X0}k_{X0} - hf_{XE}k_{XE}) = \frac{E_M}{4\pi R}. \tag{3}$$

By proposition (1), photons are carried by gravitons, therefore gravitational redshift of photons is representative of the redshift of gravitons. It follows that frequency and wavenumber of gravitons are related by: $f_X = ck_X$, and the equation can be rearranged to give

$$\frac{f_{XE}}{f_{X0}} = \sqrt{1 - \frac{E_M}{4\pi R h f_{X0} k_{X0}}}. \tag{4}$$

From special relativity: $E_M = Mc^2$ and, after substitution, the equation is now of similar form to the gravitational redshift of general relativity. The two equations become equivalent if the following holds true

$$\frac{Mc^2}{4\pi h R f_{X0} k_{X0}} = \frac{2GM}{c^2 R}. \tag{5}$$

This relation simplifies to the equation which unifies the key constants of physics

$$G = \frac{c^4}{8\pi h f_{X0} k_{X0}}. \tag{6}$$

It follows therefore that the gravitational constant G is a function of the properties of the spacetime particle. From now onwards, let equation (6) be called the unification equation. By substituting values of h and c into the unification equation, we can calculate the frequency of incident gravitons: $f_{X0} = 1.48 \times 10^{42}$ s^{-1}, the wavenumber of incident gravitons: $k_{X0} = 4.92 \times 10^{33}$ m^{-1} and the force of incident gravitons: $hf_{X0}k_{X0} = 4.85 \times 10^{42}$ Jm^{-1}. These values are of similar magnitude to those of the Planck scale, but here the values have been derived by using the mechanism of spacetime, while the Planck quantities have been obtained by dimensional analysis alone. The next step is to determine the energy of the graviton then test the correctness of our approach as well as explore the implications for established theories.

4. The Energy of Incident Gravitons

The energy of incident gravitons depends on whether a body has kinetic energy or is being influenced by the effects of a mass, because these situations may affect the measured frequency of incident gravitons. For example, a body which is held stationary with respect to the surface of a ponderous mass, receives redshifted gravitons (i.e., gravitationally redshifted gravitons) from that direction. However, a free-falling body is following a geodesic and the frequency f_{X0} of incident gravitons is not gravitationally redshifted. These concepts will be explained but first of all we are interested in the energy of incident gravitons from the perspective of a freely falling body (and leave the other situations to later). This energy can be obtained directly by substituting $f_{X0} = 1.42 \times 10^{42}$ s^{-1} into $E_X = hf_{X0}$ to give: $E_{X0} = 9.41 \times 10^8$ J or, in other units, $E_{X0} = 6.12 \times 10^{18}$ GeV [13, 14].

Thus, the graviton is a very high-energy particle indeed. This claim contrasts with current thinking and may seem wrong at first sight. We have already mentioned that many consider the energy of the graviton is so small it will never be detected, while others have said that if the graviton has mass, then its wavelength is likely to span the solar system [18]. In spite of these divergent views, this paper will show that the calculated value for the energy of the graviton is consistent with the important roles that the spacetime-particle must play.

To begin, the recent detection of gravitational waves [8, 10] demonstrates that the particle which conveys curvature of spacetime is at c, so the graviton must have zero mass. Next, curvature of spacetime affects every quantum particle in precise ways, so once we admit that the graviton is the field particle of spacetime, it follows that the wavelength of the graviton must be smaller than the wavelength of the smallest quantum particle.

This reasoning is logical because if the graviton is likened to a probe, it must have a smaller wavelength than the object with which it can interact. For example, the internal structures of bacteria and cellular organelles are generally much smaller than 1×10^{-6} m in diameter and the details of these structures cannot be resolved by visible light with a wavelength of 0.4×10^{-6} m. Instead, electron microscopes are used as the probe for this type of work because electrons have a much smaller wavelength than organelles and cellular material. Hence, for a spacetime-particle to be able to accurately interact in a consistent manner with every quantum particle in the Universe, its wavelength must be smaller than that of every other particle. It follows that the energy of the graviton must be the highest of all quantum particles.

These considerations imply that general relativity should be reinterpreted in terms of the high-energy graviton. In general relativity the spacetime constant is given by: $\kappa = \frac{8\pi G}{c^4}$, however the unification equation (6) reveals that this constant should be expressed in terms of the force of incident gravitons rather than G, as follows: $\kappa = \frac{1}{hf_{X0}k_{X0}}$. Consequently, the Einstein equation of general relativity can be expressed in terms of the mechanism of spacetime, as follows

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{1}{hf_{X0}k_{X0}} T_{\alpha\beta}. \qquad (7)$$

In this form the Einstein equation describes the effect of matter on the spacetime provided by incident gravitons. Accordingly, curvature of spacetime is caused by the effects of factors such as mass and motion on the frequency and wavenumber of incident gravitons, as revealed by the derivation of gravitational redshift (1).

In other words, mass and motion cause changes in gravitons which neighboring bodies experience as curved spacetime.

Based on this new way of interpreting the Einstein equation it is possible to consider the concept of spacetime in the void of empty space. In this type of region there is no mass, which means that the energy density ϵ_{X0} of a free point in the void is given by: $\epsilon_{X0} = hf_{X0}{k_{X0}}^3 = 1.17 \times 10^{110}$ Jm^{-3}. By a similar calculation, it can be shown that the magnitude of the cosmological pressure p_{X0} for a free point is the same as ϵ_{X0}.

Furthermore, the existence of a cosmological fluid of high-energy gravitons does not, by itself, produce curvature of spacetime or cause a vacuum catastrophe. That idea is a misunderstanding of the mechanism by which the graviton provides spacetime. The next section will show that the value which has been calculated for the energy density for a free-falling body, has important consequence for the cosmological model.

5. The Expanding Universe

The derivation of the unification equation (6) was based on changes to gravitons which occur during encounters with mass and, at first glance, it would seem that as the Universe evolves, these ongoing encounters should cause a decrease in the frequency and wavenumber of gravitons in much the same way as cosmic background radiation is redshifted. However, according to the derivation of the unification equation and the section above, the frequency and wavenumber of incident gravitons are constants for freely falling bodies. In this section this apparent paradox will be clarified and, in the process, the mechanism driving the expansion of the Universe will be revealed.

As described in the previous section and in proposition (2), encounters between gravitons and a mass cause the emitted gravitons to be redshifted and scattered. As a result, a body which is at a fixed distance from a ponderous mass, experiences curvature of spacetime due to the reduced intensity of redshifted gravitons from the direction of the mass [13, 14, 17]. However, if a body is released and falls freely towards the ponderous mass, the incident gravitons are not redshifted. Instead, incident gravitons are Doppler shifted by an extent which balances the effect of gravitational redshift. For example, a body held at the surface of the Sun receives gravitons (and photons) from the Sun which are redshifted by: $\frac{f_{XE}}{f_{X0}} = 0.999997879$, according to gravitational redshift of equation (1). However, a freely falling body does not experience the effect of this redshift because, based on the mechanism which is being proposed, the gravitational redshift of incident gravitons from the direction of the ponderous mass is countered by the Doppler blueshift of the falling body.

Also, it appears from proposition (2) that the redshift of gravitons should result in steady decreases in the cosmological pressure p_{X0} and cosmological energy density ϵ_{X0} as the Universe evolves. However for a freely falling body, gravitational redshift of incident gravitons is negated by Doppler shift, so p_{X0} and ϵ_{X0} remain constants for free bodies. Furthermore, given that the Universe consists of an immeasurable number of stellar masses which are causing an ongoing redshift and scattering of gravitons, it follows that these effects are causing the observed expansion of the Universe. In this expansion, bodies free-fall towards the incident redshifted gravitons such that the Doppler blueshift precisely counters the gravitational redshift. Consequently, the frequency f_{X0} and wavenumber k_{X0} of incident gravitons, as well as the cosmological values of ϵ_{X0} and p_{X0} remain constant for freely falling bodies during the evolution of the Universe.

This explanation is consistent with the observed Hubble recession velocity, whereby freely falling bodies receive cosmic background radiation which is Doppler redshifted. According to our explanation, at any period in the Universe's history, bodies are falling towards incident gravitons, which means that these bodies are expanding away from the photons emitted in the past. Measurements reveal that the Hubble recession velocity has been increasing in recent epochs [8, 10] and this acceleration can be explained by the mechanism proposed here.

In the evolving Universe the density of stellar bodies (i.e., stars, galaxies and larger structures) has been increasing. Accordingly, these changes in density and motion of stellar bodies should produce increased scattering and gravitational redshift of gravitons, which produce the increased rate of expansion. The mechanism which underpins the process is summarized by the unification equation (6) which succinctly describes the enduring relationship between the properties of the graviton and the impact that this relationship has on the structure of the Universe [13, 14, 17].

6. Diffraction Patterns of Gravitons

In order to prove that the graviton is the quantum field particle of spacetime, ideally the energy of the graviton should be measured in a suitable laboratory. However the energy of the graviton, as calculated above, is more than 10^7 times greater than the energy of the most powerful cosmic rays, and beyond the detection of instruments of today. Hence, direct measurement of the graviton is not possible. Instead, the existence of the graviton must be demonstrated by an indirect approach, whereby predictions are made and tested. This section reports on two predictions and the results of empirical measurements which support the proposition that the graviton is the spacetime-particle. The work reveals how scattering produces diffraction patterns of gravitons and explains cosmological effects which have been attributed to dark matter.

Suppose a set of incident gravitons encounters a mass M and, as a result, the emitted gravitons are redshifted and scattered. The Compton equation can be used to give the scattering angle ϕ of gravitons and the change in wavelength $\delta\lambda_X$ of gravitons, as follows

$$\delta\lambda_X = \lambda_{XE} - \lambda_{X0} = \frac{h}{Mc}(1 - \cos\phi). \tag{8}$$

This equation can be simplified by applying the power series to $\cos\phi$ (and assuming for weak fields that the angle for high-energy particles is small enough that magnitudes of fourth order or more can be ignored) to give

$$\delta\lambda_X = \frac{h\phi^2}{2Mc}. \tag{9}$$

Also, the change in wavelength of gravitons $\delta\lambda_X$ can be determined from the gravitational redshift as follows

$$(hf_{X0}k_{X0} - hf_{XE}k_{XE}) = \frac{Mc^2}{4\pi R}. \tag{10}$$

This equation can be modified by using the relationship: $f_X = ck_X$, and rearranging to give

$$(\frac{\lambda_{X0}}{\lambda_{XE}})^2 = (1 - \frac{Mc^2}{4\pi Rhf_{X0}k_{X0}}). \tag{11}$$

By using: $\frac{\lambda_{X0}}{\lambda_{XE}} = (1 - \frac{\lambda_X}{\delta\lambda_{XE}})$ and the binomial theorem for weak fields, the change in wavelength of gravitons is given by

$$\delta\lambda_X \approx \frac{\lambda_{XE}Mc^2}{8\pi Rhf_{X0}k_{X0}}. \tag{12}$$

The scattering angle can now be obtained by equating (9) and (12), and simplifying to give

$$\phi = \sqrt{\frac{M^2c^3}{4\pi Rh^2 f_{X0}k_{X0}k_{XE}}}. \tag{13}$$

This equation reveals that hydrogen nuclei should scatter gravitons by $\phi = 1.8 \times 10^{-29}$ radian and hydrogen atoms should scatter gravitons by $\phi = 8.5 \times 10^{-32}$ radian [14, 17]. This result can be tested and, furthermore, used to explain the effects of scattering on the dynamics of galaxies and larger structures.

One effect of scattering of gravitons should be the propagation of diffraction patterns of gravitons. This quantum phenomenon, which is not included in general relativity, can explain observations which have previously been attributed to dark matter [13, 14, 17].

To start with, consider a set of gravitons which encounters a free mass (or body). The emitted gravitons are scattered and arrive simultaneously at the surface of a sphere of radius R centered on the mass. Now consider gravitons encountering two similar masses. In this case the emitted gravitons are not spread evenly on the surface of a sphere centered on the midpoint between the masses. Instead, the gravitons emitted by one mass along the direct line between the masses, are scattered away from the line by the other mass. As a result these emitted gravitons do not arrive at two circular regions (of radius $R \sin\phi$) centered on the line joining the two masses. These gravitons are scattered to the circular rings of radius $R \sin\phi$. If the number of masses along a straight line is increased, the pattern of emitted gravitons on the surface of the sphere, is reinforced.

Next, let the number of masses (or particles) increase to form a three dimensional structure, much like a model star. Now the gravitons are scattered by planes of masses within the star and, as a result, gravitons are emitted which form a diffraction pattern that radiates from the star at c. Thus, scattering of gravitons produces regions of high density and regions of low density that emanate from the star to the surface of a sphere of radius R centered on the stellar body. The situation is analogous to Bragg's equation for the scattering of X-rays by crystals. However, here we are predicting that scattering of gravitons produces diffraction patterns of gravitons which have the unique effect of altering the spacetime of distant bodies.

Based on knowledge of diffraction patterns in general, it is logical to predict that a diffraction pattern of gravitons emanating from a star should have fringes with sharp diffraction maxima and broad diffraction minima. From trigonometry we can predict that each fringe of maxima and minima spans a regions of $R \sin\phi$. So, at a critical distance of d_c, an atom of radius r_a should be able to just fit within a diffraction minimum, as given by

$$d_c = \frac{r_a}{\sin\phi}. \tag{14}$$

This equation can be used to predict the distance at which diffraction patterns of gravitons radiating from one stellar body, influence the spacetime of masses (i.e., atoms) in another stellar body. For example, stars consisting of hydrogen should emanate diffraction patterns due to scattering of gravitons by hydrogen nuclei, with a scattering angle of $\phi = 1.8 \times 10^{-29}$ radian as calculated above from equation (13). So for atoms of radius $r_a = 0.5 \times 10^{-10}$ m in a stellar body, equation (14) gives a critical distance of $d_c = 2.8 \times 10^{18}$ m (or 0.090 kpc). This result implies that when stellar bodies are separated by this distance, their atoms are capable of just fitting within the diffraction minima of gravitons emanating from these bodies.

For stellar bodies that are closer than the critical distance, each atom within the body should receive multiple fringes of maxima and minima, and so the diffraction pattern of gravitons should have no net effect on these bodies. For example in our solar system the planets and Sun are many orders of magnitude closer than the critical distance, which means that the diffraction patterns of gravitons emanating from the Sun do not affect the motion of the planets. Thus the planets of the solar system orbit the Sun according to Einstein equation of general relativity alone.

In contrast, for two stars at the critical distance of approximately 0.090 kpc (as calculated above), the atoms within these bodies may start to be influenced by the diffraction pattern of gravitons. If the separation of the stars is a few orders of magnitude greater than the critical distance, the influence of diffraction minima dominates the geodesic of atoms in the bodies. As a result, the atoms in diffraction minima free-fall at a rate which is greater than that due to general relativity, and each star's orbital speed is greater than predicted by established theory. For separating distances which are considerably larger than the critical distance, the spacetime of orbiting stellar bodies should depend on the effects of the diffraction patterns of gravitons rather than the Einstein equation of general relativity [13, 14, 16, 17].

An examination of 62 galactic rotation curves reveals that the deviation from Newtonian mechanics occurs at an orbital distance of 0.1 kpc to 1 kpc [13, 14]. This range of empirical values concurs with the value for the critical distance of 0.090 kpc predicted above for model stars. That is to say, the predicted and calculated values for the critical distance are of similar magnitude. We have predicted that as the distance between stellar bodies increases well beyond the critical distance, the effect of diffraction minima should become the main driver of orbital speed. This prediction is consistent with the observation of flat rotation curves. Also, the spherical nature of diffraction patterns in general is consistent with the spherical galactic halos that surround galactic disks. Furthermore, the propagation of the Milky Way's diffraction patterns beyond the galactic disk can explain the observed gravitational interactions between galaxies within the local group of galaxies, and this concept has profound implications for the large-scale structure of the Universe. The next step is to demonstrate that scattering of gravitons can predict the orbital speeds of bodies in galactic halos.

Traditionally the orbital speed of stellar bodies is calculated by using either Newton's laws or the Einstein equation of general relativity. Accordingly, at the critical distance of about 0.1 kpc to 1.0 kpc, a body should orbit a Sun-like star with a speed of 6.7 ms^{-1} to 2.1 ms^{-1}. Also, traditional theory predicts that as the radius of the orbit increases beyond the critical distance, the speed will decrease by $\frac{1}{R}$. However an examination of rotation curves of 62 galaxies reveals that at distances beyond the galactic disk, the orbital speed averages 1.4×10^5 ms^{-1} with a range of 0.1×10^5 ms^{-1} to 5.0×10^5 ms^{-1} [13, 14]. These impressive speeds can be explained by determining the effect of diffraction patterns of gravitons on orbiting bodies, as follows.

The energy density in a diffraction minimum is calculated by taking into account the reduction in energy density brought about by the lack of redshifted gravitons coming from the direction of a ponderous mass [14]. Importantly, the energy of gravitons is immense, and ponderous masses such as the Sun cause a redshift which fractionally reduces this energy (as shown above). Therefore the absence of gravitons (in a diffraction minimum) causes a much greater effect on energy density than the redshift of gravitons. As a result, the equation for the change in energy density due to a diffraction minimum, is dominated by terms for the energy of the missing gravitons. The energy of the missing redshifted gravitons depends on: $\frac{M}{R_M}$, where R_M is the radius of the ponderous mass M. Also, for a body to be in a stationary orbit, the change in energy density due to a diffraction minimum must be twice the change in energy density due to the Doppler effect of orbital motion. These factors mean that for distances greater than the critical distance, the speed v of bodies orbiting non-relativistic masses is given by

$$v^2 = \frac{Mc^2}{8\pi R_M h f_{X0} k_{X0}} = \frac{GM}{R_M}. \tag{15}$$

This equation highlights a remarkable consequence of diffraction patterns of gravitons. It predicts that a body which is much more than 10^{19} m (i.e., 1 kpc) from a stellar mass M, will orbit at a speed that is many orders of magnitude greater than predicted by general relativity.

This effect of diffraction patterns of gravitons can be tested by considering the orbital speed of bodies in orbit around four model stars with the same mass as the Sun. Bodies further than the critical distance (i.e., $\gg d_c$), should orbit a red giant at 3.6×10^4 ms^{-1}, the Sun at 4.5×10^5 ms^{-1}, a white dwarf at 4.5×10^6 ms^{-1} and a neutron star at more than 3.7×10^6 ms^{-1} [14]. This range of predicted speeds is comparable to the aforementioned empirical data for the orbital speeds of galactic rotation curves [14].

The work shows that diffraction patterns of gravitons produce microscopic changes in the spacetime, thus giving bodies which are beyond the critical distance, orbital speeds much greater than predicted by general relativity. The evidence supports the predicted effects of diffraction patterns of gravitons. Thus, scattering of the spacetime-particle is the mechanism which can explain galactic halos, flat rotation curves and the intergalactic structure of the Universe. As a result the *ad hoc* invention of dark matter is unnecessary. At this point we leave the analysis of the cosmological effects of gravitons to now consider a mathematical approach to the graviton.

7. Mathematically Representing Gravitons

Everyday experiences suggest that space and time are somewhat different, however the theory of special relativity reveals that there is a connection between these two quantities, which is highlighted by the term spacetime. This paper supports the concept of spacetime and extends relativity by proposing that the graviton gives space and time in equal measure. Accordingly, the ratio of space to time is a constant c and this property is fundamental to representing points in spacetime.

Based on these considerations, we are led to proposing that the graviton performs two different mathematical operations (i.e., a dual mechanism) for constructing the dimensions of three-dimensional space and one-dimensional time. It also follows that the orthogonality of vector space is a property of gravitons. This dual mechanism of gravitons, which may be expected for a spin–2 boson, can be represented as an ordered pair or, more conveniently, as a complex number. For example, an incident graviton at a point can be represented by (x, ct) or $(x + ict)$, where x is a length of distance with direction along the x axis, while ct is a length of time. These considerations reveal how the graviton provides a foundation which will be used in the remainder of this paper to develop a mathematical structure for representing the actions of gravitons.

Each point in the Universe encounters sets of incident gravitons and, due to the dual mechanism, a point is represented by complex vector space with three complex numbers. Gravitons link points in complex vector space, though links are more than vectors. Links, for want of a better word, actually provide the connections in spacetime while vectors exist in the structure that is provided by the links. So let one point define the origin, $\vec{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and let a graviton link it to another point. This link in spacetime is given by: $0\vec{Q} = \begin{pmatrix} x + \mathrm{i}ct_x \\ y + \mathrm{i}ct_y \\ z + \mathrm{i}ct_z \end{pmatrix}$. The squared length (i.e., norm or magnitude) of this link is obtained by the inner product as follows

$$|0\vec{Q}|^2 = \langle 0\vec{Q}|0\vec{Q}\rangle = (x^2 + y^2 + z^2) + c^2(t_x^2 + t_y^2 + t_z^2). \tag{16}$$

Gravitons provide space and time in equal measure, hence the magnitude of the length in space l equals the magnitude of the length in time ct. Accordingly, the inner product and the length of the link are obtained as follows

$$|0\vec{Q}|^2 = l^2 + (ct)^2 = 2(ct)^2, \qquad |0\vec{Q}| = \sqrt{2}ct. \tag{17}$$

Next project the link onto two orthogonal subspaces, with equal length of $|ct|$. For example, in subspaces L and $L_\perp$, which contain the x and y axes respectively, the projected links have squared lengths of $|0\vec{Q}_L|^2 = (ct)^2$ and $|0\vec{Q}_{L_\perp}|^2 = (ct)^2$. Hence, in $\mathbb{C}^2$ space the coordinates of points contained within the orthogonal subspaces are represented by: $0\vec{Q}_L = \begin{pmatrix} x + \mathrm{i}ct_x \\ 0 \end{pmatrix}$ and $0\vec{Q}_{L_\perp} = \begin{pmatrix} 0 \\ y + \mathrm{i}ct_y \end{pmatrix}$. These coordinates can be normalized by dividing by the norm (i.e., by dividing by $|ct|$).

With this introduction it is becoming clear that the mathematical structure which is being constructed for representing gravitons, is adopting mathematics from quantum mechanics. This foundation will be explored later but, before that, the next section will demonstrate the usefulness of this mathematics for obtaining some of the key equations of special relativity.

8. Special Relativity with Gravitons

Consider a particle (or a theoretical point) with kinetic energy moving uniformly at u within a subspace in a direction which is labeled the x axis (i.e., $y = 0$). In this process the particle encounters gravitons which carry it and provide it with spacetime. In $\mathbb{C}^2$ space the inner product of the link from the origin to the particle is equal to $(ct)^2$ and given by

$$|0\vec{Q}_L|^2 = \begin{pmatrix} x - \mathrm{i}ct_x \\ 0 \end{pmatrix} \begin{pmatrix} x + \mathrm{i}ct_x \\ 0 \end{pmatrix} = x^2 + (ct_x)^2 = (ct)^2. \tag{18}$$

The particle moves at u along the x axis, so $|x| = |ut|$. This value can be substituted into equation (18) to obtain the length of time t_x of the moving particle at x relative to the length of time t at rest (i.e., at $\vec{0}$). The equation can be rearranged and simplified by including Einstein's relativity factor: $\gamma = \frac{1}{1-\frac{u^2}{c^2}}$, to give the equation of time dilation of special relativity, as follows

$$t^2(1 - \frac{u^2}{c^2}) = {t_x}^2, \qquad t = \gamma t_x. \tag{19}$$

Similarly, length contraction can be obtained by first determining the squared length of time $t^2 = \frac{xc}{c^2-u^2}^2$ for the particle to move a distance along the x axis, then inserting the length of time into the inner product [16].

Next, the Lorentz boosts of special relativity can be obtained by considering two subspaces containing the x axis. Let a point which is at rest on the x axis of one subspace (i.e., $0\vec{Q}_{Lk}$) move at $+u$ along the x axis in the other subspace (i.e., $0\vec{Q}_{Lj}$). From the perspective of the latter subspace, the x coordinates are related by: $x_j = x_k + ut_j$ or $x_k = x_j - ut_j$, and from the perspective of $0\vec{Q}_{Lk}$ the x axis is at rest, so $t_{xk} = t_k$. Now dividing x_k by c and rearranging gives the following

$$t_{xk} = \frac{x_k}{c} = \frac{x_j - ut_j}{c}, \qquad t_{xk} = x_j - \frac{ux_j}{c^2}. \tag{20}$$

The length of time given by equation (20) can be substituted into the inner product to obtain the following

$$\begin{aligned} \langle 0\vec{Q}_{Lk}|0\vec{Q}_{Lk}\rangle &= x_k^2 + (ct_{xk})^2 = (ct_k)^2 \\ (ct_k)^2 &= (-ut_k)^2 + c^2(t_j - \frac{ux_j}{c^2})^2 \\ {t_k}^2 &= \gamma^2(t_j - \frac{ux_j}{c^2})^2 \Rightarrow t_k = \gamma(t_j - \frac{ux_j}{c^2}). \end{aligned} \tag{21}$$

This equation is the Lorentz coordinate transformation equation (i.e., the Lorentz boost) of the time of a point on a moving x axis. By a similar process, all Lorentz coordinate transformation equations can be obtained.

Furthermore, the inner product of the projected subspace containing the x axis, can be used to obtain Minkowski's spacetime-interval. Consider a subspace $(0\vec{Q}_{Lj})$, in $\mathbb{C}^2$ space contains the x axis. The squared length of the inner product is given by

$$\langle 0\vec{Q}_{Lj}|0\vec{Q}_{Lj}\rangle = x_j^2 + (ct_{xj})^2 = (ct_j)^2, \qquad (ct_{xj})^2 = (ct_j)^2 - {x_j}^2. \tag{22}$$

The left side of this equation represents the squared length of time of the moving particle (or moving point) at x, and this quantity is independent of the speed of the particle. The right side of the equation represents the difference in the squared lengths of time and space of points at rest in the projected subspace, and the relative magnitude of these quantities does depend on the speed of the particle. Equation (22) can be generalized by considering changes in lengths in $\mathbb{C}^3$ space and rotations to obtain Minkowski's spacetime interval

$$(\mathrm{d}s)^2 = (ct)^2 - (\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2). \tag{23}$$

Thus by the derivations in this section, we see that the graviton provides the mechanism which underpins the concept of Minkowski spacetime. In obtaining these equations of special relativity, complex vector space has been used to represent points in spacetime, and the squared length between points is a real number which is obtained via the inner product. In contrast, the mathematics of special relativity does not use complex numbers to represent points in spacetime, instead it uses real numbers for coordinates to obtain the spacetime-interval. Similarly, the mathematics of general relativity does not use complex vector space, but instead relies on real coordinates and squared metrics to give squared lengths in spacetime. Thus relativity is able to overlook the quantum nature of spacetime. Next we continue to use the mathematical structure of complex vector space to explain some key aspects of quantum theory.

9. Euler's Formula

According to equation (18) a point on the x axis in $\mathbb{C}^2$ space can be represented by the projected subspace containing the x axis, as given by: $0\vec{Q}_L = \begin{pmatrix} x + \mathrm{i}ct_x \\ 0 \end{pmatrix}$, and the length of the link is obtained from the inner product: $(ct)^2 = x^2 + (ct_x)^2$, where x is the length of space from the origin and ct_x is the length of time at x (and $y = 0$). Alternatively, the point can be represented by considering the ratio of lengths of space and time in the subspace [16]. A point moving along the x axis ($y = 0$) in the projected subspace $0\vec{Q}_L$ is located at x at the time given by a circle of radius ct_x. In other words, a right angle triangle can be constructed which has hypotenuse of ct, base of x and height of ct_x, and this triangle rotates about the x axis. This construction occurs because the gravitons which radiate from the origin form a set, and a subset of that set provides the projected subspace which includes the point at x.

Firstly, the ratio for length of space x to length of the subspace ct provides the cosine ratio as follows: $\cos\phi = \frac{x}{ct}$, and reorganizing gives: $x = ct\cos\phi$. Next, the ratio of the length of time ct_x of the point at x to length of subspace ct provides the ratio: $\sin\phi = \frac{ct_x}{ct}$, and rearranging gives: $ct_x = ct\sin\phi$. These trigonometric ratios can be substituted into the coordinates of the projected subspace and the inner product to obtain the squared length of the link, as follows

$$\begin{gathered} 0\vec{Q}_L = \begin{pmatrix} x + \mathrm{i}ct_x \\ 0 \end{pmatrix} = \begin{pmatrix} ct\cos\phi + \mathrm{i}ct\sin\phi \\ 0 \end{pmatrix} \\ \langle 0\vec{Q}_L | 0\vec{Q}_L \rangle = (ct\cos\phi)^2 + (ct\sin\phi)^2 = (ct)^2. \end{gathered} \tag{24}$$

Equation (24) demonstrates the two equivalent ways of representing points in spacetime. The top row of each matrix describing the complex number z as follows

$$z = (x + \mathrm{i}ct_x) = ct(\cos\phi + \mathrm{i}\sin\phi). \tag{25}$$

This relationship can be generalize to give Euler's formula

$$z = x + \mathrm{i}y = R(\cos\phi + \mathrm{i}\sin\phi) = R\exp(\mathrm{i}\phi). \tag{26}$$

Remarkably, this procedure for obtaining Euler's formula reveals that it is accurate for all velocities of the point in the projected subspace containing the x axis. That is, the equation holds good for relativistic velocities along the x axis. Furthermore, this representation of points in spacetime can be normalized by dividing by the length of the subspace ct.

In conclusion, the mathematical structure used in these sections provides a means of deriving several key physical equations. This aspect is consistent with the notion that the graviton is a fundamental particle which, by its mechanism, provides the spacetime for physics. The derivations are characteristically simple because the spacetime-particle is the foundation of physics and mathematics. We see that in order to represent the actions of the graviton, we require knowledge of quantity (complex space, numbers and algebra), structure (sets and group theory), space (geometry and trigonometry), and change (functions and calculus). Hence, from one perspective mathematics describes the actions of the graviton while, from the other perspective, the graviton provides the relationships which are the foundations of mathematics. To demonstrate the fundamental nature of the graviton, the next section will use the mathematics to explain one of the most puzzling phenomena of quantum physics.

10. Young's Two-Slit Interference

Young's two-slit experiment, which demonstrates the effect of interference on the probability of an event, has been described as representing the last remaining mystery of quantum physics because "no one has found any machinery behind the law" [4]. However, by admitting the graviton as the spacetime-particle, there is an explanation of this phenomenon [13, 16]. The explanation relies on the dual mechanism of gravitons, whereby gravitons are added as vectors to provide a body (or quantum) with space, and simultaneously added as norms (i.e., magnitudes) to provide it with time. These two operations collectively provide the body with spacetime. First the role of gravitons in the two-slit experiment will be described, then the actions of the graviton will be analyzed.

According to proposition (1) when a quantum is emitted with kinetic energy, a set of gravitons shares in carrying the quantum in the dimension of time. Thereafter, other gravitons may join or replace gravitons in the set. For instance, when carrier-gravitons encounter the barrier the quantum may be absorbed and the number in the set becomes zero. At the edges of the slits gravitons cross paths and may add to the set of carriers. Also, because massive particles cannot be at c, carrier-gravitons are continually being replaced by other gravitons. So the number of carrier-gravitons in the set plotted against spacetime would show changes in number depending on such factors.

In a typical Young's two-slit interference experiment, an emitter releases a quantum which is propagated by an initial set of incident gravitons. Thereafter the location in space of the quantum is not defined because at any instant a set of gravitons shares in carrying the quantum. Most of the carrier-gravitons encounter the barrier, so on most occasions the energy of the quantum is transferred to the barrier (and possibly re-emitted). However on some occasions, depending on the properties of the quantum and dimensions of the slit, a subset of carrier-gravitons carries the quantum in the time dimension beyond the slits.

If there are two slits, the quantum is shared by two subsets of carrier-gravitons. Also, at the slits other gravitons which encounter the edges of the slits join the two subsets of carrier-gravitons. These subsets contain gravitons which are from the direction of the emitter as well as gravitons which are from an angle to that direction (having joined in at the edges of the slits). At any instant the quantum has different times, depending on the total length of the links (i.e., pathlengths) of the carrier-gravitons. As a result of the different pathlengths of the two subsets of carrier-gravitons, the chance that the quantum is detected at a given point on a screen is determined by the difference in the two phases of the quantum.

To analyze this description in $\mathbb{C}^2$ space, let the x axis be the direct line from the slits to the perpendicular screen where the detectors are located along the y axis. The carrier-gravitons from slits 1 and 2 provide the links in spacetime for the quantum to arrive at the screen, and the sum of the links is given by $0\vec{Q} = 0\vec{Q}_1 + 0\vec{Q}_2$. This equation can be expressed as follows

$$0\vec{Q} = \begin{pmatrix} ct_1 \exp \mathrm{i}\phi_1 + ct_2 \exp \mathrm{i}\phi_2 \\ ct_1 \exp \mathrm{i}(\frac{\pi}{2} - \phi_1) + ct_2 \exp \mathrm{i}(\frac{\pi}{2} - \phi_2) \end{pmatrix}. \tag{27}$$

The inner product of $0\vec{Q}$ gives the squared length of the links provided by the subsets of carrier-gravitons which, after calculation, is as follows

$$\langle 0\vec{Q} | 0\vec{Q} \rangle = 2(ct_1)^2 + 2(ct_2)^2 + 4c^2 t_1 t_2 \cos(\phi_2 - \phi_1). \tag{28}$$

This equation is well-known in quantum mechanics because the last term reveals the interference effect [4]. It can be simplified by substituting: $\delta = (\phi_2 - \phi_1)$, where δ represents the difference in phase of the quantum when carrier-gravitons arrive at the point of detection via two pathways. A maximum value for the squared length of spacetime occurs when the two subsets of carrier-gravitons arrive with the quantum's oscillations in phase (i.e., $\delta = 0, 2\pi, 4\pi ...$), and this condition is known as constructive interference. Destructive interference occurs when the two subsets of carrier-gravitons arrive at the detector with the quantum's oscillations completely out of phase (i.e., $\delta = \pi, 3\pi ...$), and the squared length of spacetime is a minimum. This analysis reveals that the probability of detection depends on the sum of the squared length of spacetime of the quantum, which is determined by the pathlengths of the two subsets of carrier-gravitons.

Here we have based the explanation of interference and the derivation of equation (28), on the dual mechanism of the graviton. Importantly, this explanation of Young's two-slit interference experiment follows logically from propositions (1, 2) which were introduced at the start in order to define the graviton as the spacetime-particle.

11. Conclusion

The new approach to the graviton is logical because the cumulative work of Lorentz, Poincaré, Einstein, Minkowski and others, clearly demonstrates that we should think in terms of spacetime rather than gravitational interactions. This new definition provides the mechanism for spacetime which is missing from the theories of relativity and quantum mechanics. It leads to a new perspective that improves understanding of physical phenomena and does not require *ad hoc* inventions, as has occurred recently in modern physics.

The unification equation (6) has been derived and used to extend the established theories of general relativity and quantum theory. This equation has profound implications. Firstly, it reveals that the gravitational constant G is based on the quantum properties of the graviton. Also, it has been used to calculate that a freely falling body finds incident gravitons have frequency $f_{X0} = 1.48 \times 10^{42}$ s^{-1}, wavenumber $k_{X0} = 4.92 \times 10^{33}$ m^{-1} and energy of 6.12×10^{18} GeV. At first glance the magnitude of the graviton's energy may appear incredible, however it has been argued that this energy determines the cosmological structure of the Universe, and accounts for observations which are driven by 95 % of the Universe's total energy.

Furthermore, gravitational redshift should result in an expansion of the Universe and it has been argued that bodies freely fall towards redshifted incident gravitons, such that f_{X0}, k_{X0}, cosmological energy density and cosmological pressure are maintained as constants for freely falling bodies (or particles or points) during the evolution of the Universe. Scattering should produce diffraction patterns of gravitons and in this paper equations have been derived which have been used to predict scattering angles as well as the orbital speeds of distant bodies. Cosmological observations support these predictions and it is concluded that diffraction patterns of gravitons account for flat rotation curves, spherical galactic halos and intergalactic interactions.

This paper has highlighted the need to use a mathematical structure based on complex vector space in order to represent and understand the actions of gravitons. This approach has been used to obtain some key equations, such as Einstein's time dilation, Lorentz boosts, Minkowski's spacetime interval, and Euler's formula. Also, Young's two-slit interference has been explained with gravitons. So by defining the graviton as the quantum field particle of spacetime, the work reveals that the graviton is the mechanism which provides the foundation for the beautiful relationships that exist between physics and mathematics.

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