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# CONSTRUCTION OF THE SHELL IN NONSYMMETRIC GRAVITY

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**Abstract.** We examine the self-gravitating spherical shell in the fully nonlinear and nonsymmetric theory of the gravity. We argue that the hyperdense static finally collapsed object could not be made of any known form of matter. Also we observed that if the radius of the shell is sufficiently small then the antisymmetric part of the energy-momentum tensor exceeds its symmetric part. It seems to violate the natural physical conditions.

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## 1. Introduction

Almost from the beginning of the theory of General Relativity many researches noticed the problem connected with existence of the singularities. There are at least three main ways to overcome this problem. In the most conservative approach it is presumed that the evolution of the system is well defined outside of the event horizons. The validity of this approach needs the proof of the Cosmic Censorship Conjecture. In the second approach it is presumed that all singularities disappear at the quantum level of the gravity theory (see for example [19], [12], [17]). The last approach is based on the conviction that there is possibility to construct classical gravity theory that is free of singularities. In the third strategy the potential alternative theory of gravity must contain General Relativity as the weak field limit. This requirement is imposed on new proposals of gravity theories because in the weak field regime the agreement of the Einstein theory with experiment is almost perfect.

One of proposals of such a theory is based on non-Riemannian geometry [9], in which the metric tensor $g_{\mu\nu}$ can be split into symmetric and skew-symmetric part. From mathematical point of view non-Riemannian geometry enables to circumvent assumptions of the Hawking and Penrose singularity theorems [10].

On the other hand the physical motivation is the following: if one allows the metric tensor to posses symmetric $g_{(\mu\nu)}$ and skew-symmetric part $g_{[\mu\nu]}$ as well

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]} \tag{1}$$

then the distances are not affected by the antisymmetric part of the metric tensor i.e.,

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = g_{(\mu\nu)}\mathrm{d}x^\mu \mathrm{d}x^\nu. \tag{2}$$

Moreover, the skew-symmetric part affects the determinant i.e., volume element of the space. In these conditions there is a hope that matter in the sphere of fixed radius (identical with the radius of the sphere in General Relativity Theory) can form enough volume to prevent matter from formation of the black hole.

Historically the motivation was different, first nonsymmetric theory was formulated by Einstein [7] and then developed by Schrödinger [18] and Straus [8]. Einstein was motivated by desire of constructing a Unified Field Theory of classical gravity and electrodynamics. The interest in nonsymmetric gravity has been restored by Moffat [13] (and references therein).

In 1993 Damour et al [5] showed that wide class of Nonsymmetric Gravity Theories (NGT) posses serious consistency problems. They proved that a generic nonsymmetric model is not free of negative energy excitations (“ghosts”) and posses some algebraic inconsistencies.

Finally, consistent gravity theory with nonsymmetric metric tensor has been formulated in paper [14].

A static and spherically symmetric vacuous solution of NGT has been investigated by Cornish and Moffat [3]. This solution depends on two integration constants $m$ and $s$. The first constant $m$ denotes the total mass of the system. The second (dimensionless) constant $s$ measures a contribution of the skew part of the metric. For $s = 0$ a space-time is described by Schwarzschild geometry. Moreover, for $r > 2m$ and small $s$ the solution is also approximated by the Schwarzschild metric. On the other hand, in the strong gravitational field regime i.e., for $r < 2m$, the limit $s \to 0$ is nonanalytic. Nevertheless in this regime the configuration is regular and does not contain a black hole horizon. On the base of this solution one can expect that such behavior is generic for NGT and the theory is free of singularities. The linearized version of NGT has been analyzed by Burko and Ori [1]. They analyzed the dynamical behaviour of the skew field linearized around the Schwarzschild background. In this approximation the skew field can not be static and therefore the formation of a black hole with singularity can be anticipated. Cornish and

Moffat suggested that the conclusion of paper [1] is a result of the approximation and will not appear in the fully nonlinear theory[2].

Moreover the attempt to analyze the full picture of the gravitational collapse in NGT has been undertaken in paper [15]. The author studied the collapse of the spherically symmetric dust cloud. He concluded that the dust cloud does not collapse to a black hole but forms “finally collapsed object” which achieves some state of equilibrium. Due to simplifying assumptions about the nature of the skew-symmetric part of the field equations and because of used approximations this result also can not be conclusive. Extended studies of the spherically symmetric collapse in NGT were also performed in paper [4], where numerical analysis indicates dynamical instability of the system.

We analyze the spherical shell in the fully nonlinear and nonsymmetric gravity theory. We concentrate our analysis on the most interesting case of a small compact system. The analysis shows that the static configuration requires existence of the huge amount of the antisymmetric component of energy-momentum tensor which breaks natural physical conditions.

This paper is organized as follows. In the next section we remain equations of the Nonsymmetric Gravity Theory. Then field equations of NGT are adopted to the spherically symmetric case. Section 3 contains the exact solution of NGT that describes spherically symmetric infinitesimally small shell. In this section we also investigate properties of the energy-momentum tensor. Last section contains remarks.

## 2. The Field Equations

The field equations of Nonsymmetric Gravity Theory have been proposed in paper [14]

$$G_{\mu\nu}(W) + \lambda g_{\mu\nu} + \frac{1}{4}\mu^2 C_{\mu\nu} + \frac{1}{2}\sigma(P_{\mu\nu} - \frac{1}{2}g_{\mu\nu}P) = 8\pi T_{\mu\nu} \tag{3}$$

$$\mathbf{g}^{[\mu\nu]}{}_{,\nu} = 3\mathbf{D}^{\mu} \tag{4}$$

$$\mathbf{g}^{\mu\nu}{}_{;\sigma} + \mathbf{g}^{\rho\nu}W^{\mu}_{\rho\sigma} + \mathbf{g}^{\mu\rho}W^{\nu}_{\sigma\rho} - \mathbf{g}^{\mu\nu}W^{\rho}_{\sigma\rho} + \frac{2}{3}\delta^{\nu}_{\sigma}\mathbf{g}^{\mu\rho}W^{\beta}_{[\rho\beta]} + \mathbf{D}^{\nu}\delta^{\mu}_{\sigma} - \mathbf{D}^{\mu}\delta^{\nu}_{\sigma} = 0 \tag{5}$$

where $W^{\mu}_{\rho\sigma}$ denotes the nonsymmetric connection. The cosmological constant is denoted by $\lambda$. The equations (3-5) contain also additional cosmological constants $\mu$ and $\sigma$ which are associated respectively with the skew part of the metric tensor $g_{[\mu\nu]}$ and the field $W_{\mu} = \frac{1}{2}(W^{\lambda}_{\mu\lambda} - W^{\lambda}_{\lambda\mu})$. We have in mind that square brackets denote antisymmetrisation and round brackets symmetrisation operation. Additionally, field equations contain the Einstein like tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R. \tag{6}$$

The other quantities present in equations of motion are defined below

$$R_{\mu\nu}(W) = W^{\beta}_{\mu\nu,\beta} - \frac{1}{2}(W^{\beta}_{\mu\beta,\nu} + W^{\beta}_{\nu\beta,\mu}) - W^{\beta}_{\alpha\nu}W^{\alpha}_{\mu\beta} + W^{\beta}_{\alpha\beta}W^{\alpha}_{\mu\nu} \tag{7}$$

$$g^{\mu\nu}g_{\sigma\nu} = g^{\nu\mu}g_{\nu\sigma} = \delta^{\mu}_{\sigma} \tag{8}$$

$$C_{\mu\nu} = g_{[\mu\nu]} + \frac{1}{2}g_{\mu\nu}g^{[\sigma\rho]}g_{[\rho\sigma]} + g^{[\sigma\rho]}g_{\mu\sigma}g_{\rho\nu} \tag{9}$$

$$P_{\mu\nu} = W_{\mu}W_{\nu}, \qquad P = g^{\mu\nu}P_{\mu\nu} \tag{10}$$

$$\mathbf{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}, \qquad \mathbf{D}^{\mu} = \frac{1}{2}\sigma\mathbf{g}^{(\mu\alpha)}W_{\alpha}. \tag{11}$$

An interesting property of this theory is the fact that the value of the parameter $\sigma$ is fixed by the requirement of self consistency [14]. It has been showed that the theory is consistent for $\sigma = -\frac{1}{3}$. Some simplification of the equation (5) can be achieved by introducing a new connection $\Gamma^{\lambda}_{\mu\nu}$

$$\Gamma^{\lambda}_{\mu\nu} = W^{\lambda}_{\mu\nu} + \frac{2}{3}\delta^{\lambda}_{\mu}W_{\nu}. \tag{12}$$

New form of the equation (5) is the following

$$\mathbf{g}^{\mu\nu}{}_{,\sigma} + \mathbf{g}^{\rho\nu}\Gamma^{\mu}_{\rho\sigma} + \mathbf{g}^{\mu\rho}\Gamma^{\nu}_{\sigma\rho} - \mathbf{g}^{\mu\nu}\Gamma^{\rho}_{(\sigma\rho)} + \mathbf{D}^{\nu}\delta^{\mu}_{\sigma} - \mathbf{D}^{\mu}\delta^{\nu}_{\sigma} = 0. \tag{13}$$

The contracted curvature tensor of the NGT can be related with Ricci-like tensor

$$R_{\mu\nu}(W) = R_{\mu\nu}(\Gamma) + \frac{2}{3}W_{[\mu,\nu]}. \tag{14}$$

Significant simplification of the field equations is achieved in the case of spherical symmetry. The most general form of the metric tensor $g_{\mu\nu}$ in NGT has been derived in article [16]

$$g_{\mu\nu} = \begin{pmatrix} \gamma & w & 0 & 0 \\ -w & -\alpha & 0 & 0 \\ 0 & 0 & -\beta & f\sin\theta \\ 0 & 0 & -f\sin\theta & -\beta\sin^2\theta \end{pmatrix}. \tag{15}$$

Moreover, the assumption of the absence of the magnetic monopole implies that $w = 0$. The skew symmetric part of the metric in this case is therefore described by a single function $f$. In this case the fields $W_{\mu}$, $P_{\mu\nu}$ and $\mathbf{D}^{\mu}$ become equal to zero [14] and therefore

$$W^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}, \qquad P_{\mu\nu} = 0, \qquad \mathbf{D}^{\mu} = 0. \tag{16}$$

If we additionally put cosmological constants $\lambda = \mu = 0$ then the system of equations (3-11) acquire a more familiar form

$$G_{\mu\nu}(\Gamma) = 8\pi T_{\mu\nu}, \qquad \mathbf{g}^{[\mu\nu]}{}_{,\nu} = 0 \tag{17}$$

$$g_{\mu\nu}{}_{,\sigma} - g_{\rho\nu}\Gamma^{\rho}_{\mu\sigma} - g_{\mu\rho}\Gamma^{\rho}_{\sigma\nu} = 0. \tag{18}$$

Finally, we focus our interest on static field configuration and therefore the only non vanishing components of $G_{\mu\nu}$ have the form

$$G_0{}^0 = -\frac{1}{2\alpha}A'' - \frac{1}{16\alpha}[3(A')^2 + 4B^2] + \frac{A'\alpha'}{4\alpha^2} + \frac{r^2}{r^4 + f^2} \tag{19}$$

$$G_1{}^1 = \frac{1}{16\alpha}[-(A')^2 + 4B^2] - \frac{A'\gamma'}{4\alpha\gamma} + \frac{r^2}{r^4 + f^2} \tag{20}$$

$$\begin{aligned} G_2{}^2 = G_3{}^3 = &-\frac{\gamma''}{2\alpha\gamma} + \frac{\gamma'}{4\alpha\gamma}\left(\frac{\alpha'}{\alpha} + \frac{\gamma'}{\gamma}\right) \\ &+ \frac{A'}{8\alpha}\left(\frac{\alpha'}{\alpha} - \frac{\gamma'}{\gamma}\right) - \frac{1}{4\alpha}A'' - \frac{1}{16\alpha}[(A')^2 + 4B^2] \end{aligned} \tag{21}$$

$$G_3{}^2 = -\sin^2\theta G_2{}^3 = \sin\theta\left[\frac{f}{r^4 + f^2} + \frac{B}{4\alpha}\left(\frac{\gamma'}{\gamma} - \frac{\alpha'}{\alpha} + A'\right) + \frac{B'}{2\alpha}\right] \tag{22}$$

where

$$A = \ln(r^4 + f^2), \qquad B = \frac{2fr - r^2 f'}{r^4 + f^2} \tag{23}$$

and we made the standard choice of $\beta = r^2$.

## 3. Spherically Symmetric Shell

The only known asymptotically flat solution with spherical symmetry of the Nonsymmetric Gravity Theory was found by Wyman [20]. In terms of the metric components (15), this solution can be written in the following way

$$\alpha = \frac{\mathrm{e}^{-\nu} m^2 (\nu')^2 (1 + s^2)}{[\cosh(a\nu) - \cos(b\nu)]^2}, \qquad \beta = r^2, \qquad \gamma = \mathrm{e}^{\nu} \tag{24}$$

$$f = \frac{2m^2 \mathrm{e}^{-\nu}[\sinh(a\nu)\sin(b\nu) + s(1 - \cosh(a\nu)\cos(b\nu))]}{[\cosh(a\nu) - \cos(b\nu)]^2} \tag{25}$$

where

$$a^2 = \frac{\sqrt{1 + s^2} + 1}{2}, \qquad b^2 = \frac{\sqrt{1 + s^2} - 1}{2}\cdot \tag{26}$$

Prime denotes differentiation with respect to space variable $r$. The function $\nu = \nu(r, s)$ is implicitly determined by the equation

$$\mathrm{e}^{\nu}[\cosh(a\nu) - \cos(b\nu)]^2 \frac{r^2}{2m^2} = \cosh(a\nu)\cos(b\nu) + s\sinh(a\nu)\sin(b\nu) - 1 \tag{27}$$

where $s$ is a dimensionless constant of integration.

**For small** $r$ (i.e., for $r/2m \ll 1$) and for $|s| < 1$ the solution can be expanded in radial variable. The leading terms of this expansion are the following

$$\alpha = \frac{4\xi}{s^2}\left(\frac{r}{m}\right)^2, \qquad \gamma = \xi + \frac{\xi}{2|s|}\left(\frac{r}{m}\right)^2 \tag{28}$$

$$f = m^2\left(4 - \frac{|s|\pi}{2} + s|s|\right) + \frac{|s| + s^2\pi/8}{4} r^2 \tag{29}$$

$$\xi = \exp\left(-\frac{\pi + 2s}{|s|}\right). \tag{30}$$

The formulas (28) and (30) explicitly illustrate the nonanalytic nature of the limit $s \to 0$ in the strong gravitational field regime for $r < 2m$.

Our strategy of constructing the shell solution is based on analogy to the similar problem in Einstein Theory of Gravitation (see appendix). We use two Wyman solutions for different masses and glue them on the shell. Next we analyse the components of the energy-momentum tensor on the sphere which joins the solutions. Let us notice that the Minkowski space-time (with $f = 0$) can not be an internal solution because discontinuity of $f$ produces derivatives of the delta function in ${T_\mu}^\nu$ equations (19-23).

We restrict our considerations to the most interesting case of **infinitesimally small shells**. In this case we can use functions (28-30) as components of the metric. Additionally, equations (19-22) posses scaling symmetry: $\gamma \to k\gamma$. This symmetry is used only inside the shell because solution (24-27) outside the shell could be identified with Schwarzschild solution only for $k = 1$. Hence the internal solution may be written in the form

$$\alpha_- = \frac{4\xi_-}{s_-^2}\left(\frac{r}{\mu}\right)^2, \qquad \gamma_- = k\left(\xi_- + \frac{\xi_-}{2|s_-|}\left(\frac{r}{\mu}\right)^2\right) \tag{31}$$

$$f_- = \mu^2 C_- + D_- r^2, \qquad \xi_- = \exp\left(-\frac{\pi + 2s_-}{|s_-|}\right) \tag{32}$$

$$C_- = 4 - \frac{|s_-|\pi}{2} + s_-|s_-|, \qquad D_- = \frac{|s_-| + s_-^2\pi/8}{4} \tag{33}$$

where $\mu$ is a mass-like parameter. Because we assumed that the radius $R$ of the shell is small then outside the shell we still can use formulas (28-30) for $\gamma_+, \alpha_+$ and $f_+$

$$\alpha_+ = \frac{4\xi_+}{s_+^2}\left(\frac{r}{m}\right)^2, \qquad \gamma_+ = \xi_+ + \frac{\xi_+}{2|s_+|}\left(\frac{r}{m}\right)^2 \tag{34}$$

$$f_+ = m^2 C_+ + D_+ r^2, \qquad \xi_+ = \exp\left(-\frac{\pi + 2s_+}{|s_+|}\right) \tag{35}$$

$$C_+ = 4 - \frac{|s_+|\pi}{2} + s_+|s_+|, \qquad D_+ = \frac{|s_+| + s_+^2\pi/8}{4}. \tag{36}$$

Here $m$ is the total mass of the system. The formulas (31-36) can be combined in one set of equations that describe the "global" solution

$$\gamma = \xi_+ + \frac{\xi_+}{2|s_+|}\left(\frac{\tilde{r}}{m}\right)^2 \tag{37}$$

where

$$\tilde{r}^2 = \begin{cases} 2m^2|s_+|\left(k\frac{\xi_-}{\xi_+} - 1\right) + k\frac{\xi_-}{\xi_+}\frac{|s_+|}{|s_-|}\frac{m^2}{\mu^2}r^2 & \text{for } r < R \\ r^2 & \text{for } r \geq R \end{cases} \tag{38}$$

$$k = \frac{\xi_+ + \frac{\xi_+}{2|s_+|}\left(\frac{R}{m}\right)^2}{\xi_- + \frac{\xi_-}{2|s_-|}\left(\frac{R}{\mu}\right)^2}. \tag{39}$$

This choice of $k$ ensures continuity of $\gamma$ function. The radius of the shell is denoted by $R$. The field equations are second order therefore the functions $\gamma$ and $f$ have to be $C^1$. In the equations we have the first derivative of $\alpha$ and second derivatives of $\gamma$ and $f$ which produce $\delta$ function on the shell, in the energy-momentum tensor. Functions $\alpha$ and $f$ have the form

$$\alpha = 4\frac{\xi_-}{{s_-}^2}\frac{r^2}{\mu^2} + 4\left(\frac{\xi_+}{{s_+}^2 m^2} - \frac{\xi_-}{{s_-}^2\mu^2}\right)r^2\Theta(r-R) \tag{40}$$

$$f = \begin{cases} f_- & \text{for } r < R \\ f_+ & \text{for } r \geq R \end{cases} \tag{41}$$

$$\mu^2 C_- + D_- R^2 = m^2 C_+ + D_+ R^2 \tag{42}$$

where $|s_-| < 1$, $|s_+| < 1$. and $\Theta$ denotes the step function. Equation (42) ensures continuity of the function $f$. Finally substitution of (36-41) into equations (17-23) gives the energy-momentum tensor

$$T_{\phi\theta} = -T_{\theta\phi} = \beta\sin^2\theta T_\theta^\phi + f\sin\theta T_\phi^\phi$$

which has completely antisymmetric off diagonal components.

## 4. Energy-Momentum Tensor of the Shell

At the beginning let us compare the physical "antisymmetric" component $\tau = {T_\theta}^\phi$ with the matter density $\rho = {T_0}^0$. From equations (19) and (22) we find the components of the energy-momentum tensor ${T_\mu}^\nu$

$$\rho = \frac{1}{8\pi}\left(\frac{-A''}{2\alpha} + \frac{A'\alpha'}{4\alpha^2}\right), \qquad \tau = \frac{\sin\vartheta}{8\pi}\left(\frac{B'}{2\alpha} - \frac{B\alpha'}{4\alpha^2}\right). \tag{43}$$

We analyze the case of a small shell $(R \ll 2m)$ and therefore after some algebra we get

$$\max|\frac{\tau}{\rho}| = \left|\frac{B\alpha' - 2\alpha B'}{A'\alpha' - 2\alpha A''}\right| \to \frac{1}{|2D_+ - 2(D_+ - D_-)\omega|} \tag{44}$$

where

$$\omega = \frac{1+\kappa}{1-\kappa}, \qquad \kappa = \frac{s_+^2 m^2 \xi_-}{s_-^2 \mu^2 \xi_+}. \tag{45}$$

For simplicity, let us consider small $|s_+| << 1$ limit. For $R \ll 2m$ a constant $s_-$ approaches some non zero value determined by the equation (42). Moreover, because of the exponential vanishing of $\xi_+$ in the described limit we have $\kappa \to \infty$ and $\omega \to -1$. Hence

$$\max|\frac{\tau}{\rho}| \to \frac{1}{|2D_-|} > \frac{2}{1+\pi/8} \simeq 1.44. \tag{46}$$

The last inequality is a consequence of equation (33) and $|s_-| < 1$.

## 5. Remarks

We constructed in the fully nonlinear theory static spherically symmetric solution in the form of the shell (which is located below the General Relativity horizon $R << 2m$). We have shown that the energy-momentum tensor of small static and spherically symmetric shell contains large amount of the antisymmetric component. On the other hand, for all known form of matter energy-momentum tensor is purely symmetric. Inequality (46) does not predict that antisymmetric component of $T_{\mu\nu}$ is small enough as to be unobservable and therefore it seems to indicate the violation of natural physical conditions.

The other possibility is that "finally collapsed object" is made of unknown form of matter (hidden in strong gravitational fields), which could be in principle candidate for dark matter in missing mass problem in cosmology. Unfortunately, the similar calculation performed in General Relativity (see Appendix) uniquely indicates that collapse of the shell could be stopped only by negative matter density. This result means that there is no physically sensible matter which can be used in order to construct (located below the horizon) static shell in General Relativity Theory.

Therefore we lean towards a view that also in NGT there does not exist static finally collapsed object consisting of the physically reasonable matter.

## 6. Appendix

The similar calculation can be performed in the Einstein gravity by gluing two concentric Schwarzschild solutions with different masses $\mu$ and $m$ $(m > \mu)$. In co-moving coordinates one easy finds that if the radius of the shell is constant then matter density is the following

$$\rho = \frac{m - \mu}{4\pi R(R - 2m)} \delta(r - R).$$

In the trapped region $R < 2m$ matter density becomes negative. This result show that in Einstein theory of gravity the small static shell can only be made of non-physical matter. Above result can be also obtained in the standard approach of Israel [11].

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