

## Indices (Index of Notation, General Index), 147-151

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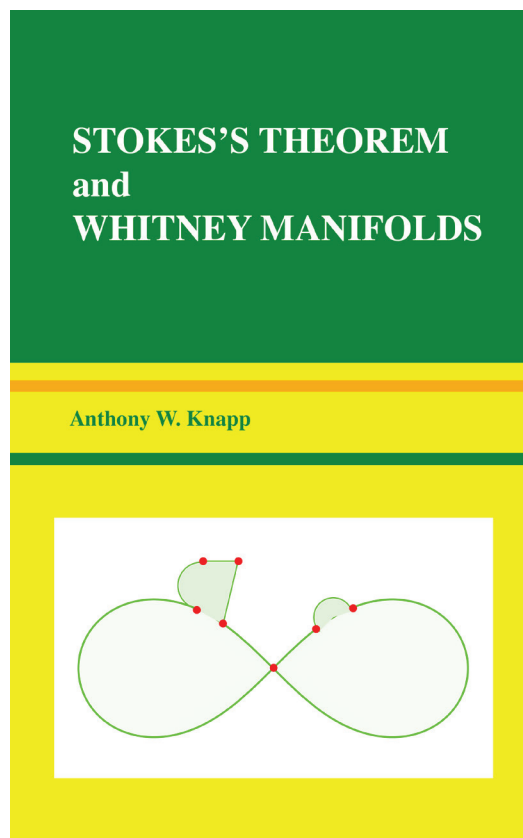
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***Stokes's Theorem and  
Whitney Manifolds:  
A Sequel to  
Basic Real Analysis***

Anthony W. Knapp

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Title: Stokes’s Theorem and Whitney Manifolds. A Sequel to *Basic Real Analysis*.

Cover: An example of a Whitney domain in two-dimensional space. The green portion is a manifold-with-boundary for which Stokes’s Theorem applies routinely. The red dots indicate exceptional points of the boundary where a Whitney condition applies that says Stokes’s Theorem extends to the whole domain.

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## INDEX OF NOTATION

This list indexes recurring symbols introduced in Chapters I through III (pages 1–125). In the list below, each piece of notation is regarded as having a key symbol. The first group consists of those items for which the key symbol is a fixed Latin letter, and the items are arranged roughly alphabetically by that key symbol. The next group consists of those items for which the key symbol is a Greek letter. The final group consists of those items for which the key symbol is a variable or a nonletter, and these are arranged by type. To locate an item below, first proceed on the assumption that the key symbol is a Latin or Greek letter; if the item does not appear to be in the list, then treat it as if its key symbol is a variable or a nonletter.

$\mathcal{A}$ , 18 $B^\delta$ , 111 $c_X(\omega)$ , 50 $C^\infty$ , 4, 63, 99 $C_{\text{com}}^\infty$ , 5, 64, 99 $\mathcal{C}_p(M)$ , 7, 64, 99 curl, 29, 76 $d$ , 28 $df$ , 11 $(df)_p$ , 11 $D$ , 9, 65, 99 $(DF)_p$ , 9, 99 $D(x, E)$ , 106 div, 29, 76 grad, 29, 76 $\mathbb{H}^m$ , 59 $\mathbb{H}_+^m$ , 59 $I'$ , 13 <b>i, j, k</b> , 75 $\mathcal{M}^\ell(E)$ , 109	<b>n</b> , 77 $N(E, \delta)$ , 111 $N_{\text{sep}}(E, \delta)$ , 111 $\mathbb{Q}^m$ , 93 $\mathbb{Q}_+^m$ , 93 $\partial\mathbb{Q}^m$ , 93 $S^n$ , 2 $S_k(M)$ , 100 $\mathfrak{S}_n$ , 18 $T(V)$ , 13 $T^n(V)$ , 13 $T_p(M)$ , 7, 64, 99 $T_p^*(M)$ , 10, 65, 99  <b>Greek</b> $\Phi^*$ , 22 $\Phi^*\omega$ , 23 $\Phi_p^\#$ , 23 $\Omega^k(M)$ , 22
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**Unary operations** $M_+$ , 61 $\partial M$ , 61 $\nabla$ , 76 $|\cdot|$ , 106**Binary operations** $\otimes$ , 13 $\wedge$ , 13 $\times$ , 76**Other symbols** $\wedge(E)$ , 13 $\wedge^n(E)$ , 13 $\tilde{\wedge}^n(E)$ , 18 $[\frac{\partial}{\partial x_j}]_p$ , 7, 64, 99 $\int_M f \omega$ , 39 $E^\delta$ , 111 $(M_\alpha, \alpha)$ , 2, 60, 95, 119 $(U, B, E)$ , 107

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