Annotated Bibliography

A (cut) Short Story:
Peter wanted to know the names of the birds.
He read a book and learned the names of the birds.
Peter wanted to learn how to swim.
He read a book and drowned.
— from E.C. Basar, R.A. Bonic, et al, *Studying Freshman Calculus*,
Lexington, MA: D.C. Heath and Co., 1976

This bibliography consists of books (and other items) that are referenced in this book, books used in the writing this book, or books that I have read that I think readers may be interested in. There is a more complete (and frequently updated) bibliography with more than 300 listings on the web at

http://www.math.cornell.edu/~dwh/biblio

The section names below correspond to a subset of the section names in the bibliography on the web.

The annotations in quotes are taken from the introductory matter, cover, or dust jacket of the book being annotated.

AT Ancient Texts

al'Khowarizmi, *Algebra*

This is the world's first algebra text. An English translation is contained in Karpinski, L.C., ed., *Robert of Chester's Latin Translation of Al'Khowarizmi's Algebra*, New York: Macmillan, 1915.

Apollonius of Perga, *Treatise on Conic Sections*, T.L. Heath, ed., New York: Dover, 1961.

This is the standard work on conic sections from the Greek world.

Apollonius of Perga, *On Cutting Off a Ratio*, E.M. Macierowski, trans., R.H. Schmidt, ed., Fairfield: The Golden Hind Press, 1987.

"An attempt to recover the original argumentation through a critical translation of the two extant medieval Arabic manuscripts."

Baudhayana, *Sulbasutram*, G. Thibaut, trans., S. Prakash & R. M. Sharma, ed., Bombay: Ram Swarup Sharma, 1968.

This is translated from the Sanskrit manual for the construction of altars. The beginning of the book contains a textbook of the geometry needed for the construction of the altars — this beginning section is apparently the oldest surviving geometry textbook. See Appendix B and Chapter 13.

Bonasoni, Paolo, *Algebra Geometrica*, R. H. Schmidt, trans., Annapolis: The Golden Hind Press, 1985.

"being the only known work of this nearly forgotten Renaissance mathematician (excepting a still unpublished treatise on the division of circles)."

Cardano, Girolamo, *The Great Art or the Rules of Algebra*, T.R. Witmer, ed., Cambridge: MIT Press, 1968.

This is the book that first describes algebraic algorithms for solving most cubic equations.

Descartes, Rene, *The Geometry of Rene Descartes*, D. Eugene, M.L. Latham, trans., New York: Dover Publications, Inc., 1954.

This the book in which Descartes develops the use of what we now call *Cartesian coordinates* for the study of curves.

Euclid, *Elements*, T.L. Heath, ed., New York: Dover, 1956.

This is the edition of Euclid's *Elements* to which one is usually referred. Heath has added a large collection of very useful historical and philosophical notes.

Euclid, *Optics*, H. E. Burton, trans., *Journal of the Optical Society of America*, vol. 35, no. 5, pp. 357–72, 1945.

This is a translation of Euclid's work that contains the elements of what we now call projective geometry.

Euclid, *Phaenomena*, in *Euclidis opera omnia*, Heinrich Menge, ed., Lipsiae: B.G. Teubneri, 1883-1916.

A work on astronomy that discusses aspects of spherical geometry.

Guthrie, Kenneth, *The Pythagorean Sourcebook and Library*, Grand Rapids: Phanes Press, 1987.

"An Anthology of Ancient Writings Which Relate to Pythagoras and Pythagorean Philosophy."

Khayyam, Omar, *Algebra*, D.S. Kasir, ed., New York: Columbia Teachers College, 1931 (and New York: A.M.S. Press, 1972).

In this book Khayyam gives geometric techniques for solving cubic equations.

Khayyam, Omar, *Risâla fî sharh mâ ashkala min musâdarât Kitâb 'Uglîdis*, A.I. Sabra, ed., Alexandria, Egypt: Al Maaref, 1961.

This is the Arabic original of Khayyam's discussions of non- Euclidean geometry. Translated in A. R. Amir-Moez, "Discussion of Difficulties in Euclid" by Omar ibn Abrahim al-Khayyami (Omar Khayyam), *Scripta Mathematica*, 24 (1958–59), pp. 275– 303.

Khayyam, Omar, a paper (no title).

Translated in A. R. Amir-Moez, "A Paper of Omar Khayyam," *Scripta Mathematica*, 26(1963), pp. 323–37. In this paper Khayyam discusses algebra in relation to geometry.

Plato, *The Collected Dialogues*, Edith Hamilton and Huntington Carns, eds., Princeton, NJ: Bollinger, 1961.

Plato discusses mathematical ideas in many of his dialogues.

Proclus, *Proclus*, *A Commentary on the First Book of Euclid's Elements*, Glenn R. Morrow, trans., Princeton: Princeton University Press, 1970.

These commentaries by Proclus (Greek, 410–85) are a source of much of our information about the thinking of mathematicians toward the end of the Greek era.

Saccheri, Girolamo, *Euclides Vindicatus*, G.B. Halsted, ed. and trans., New York: Chelsea Pub. Co., 1986.

"In this book Girolamo Saccheri set forth in 1733, for the first time ever, what amounts to the axiom systems of non-Euclidean geometry." It is not mentioned in this volume that Saccheri borrowed many ideas from Khayyam's *Risâla fî sharh mâ ashkala min musâdarât Kitâb 'Uglîdis*.

Theon of Smyrna, *Mathematics Useful for Understanding Plato*, R. and D. Lawlor, trans., San Diego: Wizards Bookshelf, 1978.

"This work appears to have been a text book intended for students who were beginning a study of the works of Plato. In its original form there were five sections: 1) Arithmetic 2) Plane Geometry 3) Stereometry (solid geometry) 4) Music 5) Astronomy. Sections 2 and 3 on Geometry have been lost while the others remain in their entirety and are presented here." The section on Astronomy contains discussions of the shape of space.

Thomas, Ivor, trans., *Selections Illustrating the History of Greek Mathematics* , Cambridge, MA: Harvard University Press, 1951.

A collection of primary sources.

CG Computers and Geometry

The Geometer's Sketchpad: Dynamic Geometry for the 21st Century, Key Curriculum Press.

A program running on Windows or Mac platforms that allows you to construct geometric drawing with points, lines, and circles and then dynamically vary constituent parts.

Richter-Gerbert, Jürgen, and Ulrich H. Kortenkamp, *Cinderella*: *The Interactive Geometry Software*, Heidelberg: Springer-Verlag, 1999.

A *Java*-based dynamic geometry software.

Taylor, Walter F., *The Geometry of Computer Graphics*, Pacific Grove, CA: Wadsworth & Brooks/Cole Advanced Books & Software, 1992.

"This book is a direct presentation of elementary analytic and projective geometry, as modeled by vectors and matrices and as applied to computer graphics."

DG Differential Geometry

Bloch, Ethan D., *A First Course in Geometric Topology and Differential Geometry*, Boston: Birkhauser, 1997.

This book contains the topological classification and differential geometry of surfaces.

Casey, James, *Exploring Curvature*, Wiesbaden: Vieweg, 1996.

A truly delightful book full of "experiments" to physically explore curvature of curves and surfaces.

do Carmo, Manfredo, *Differential Geometry of Curves and Surfaces*, Englewood Cliffs, NJ: Prentice Hall, 1976.

An undergraduate level text.

Dodson, C.T.J., and T. Poston, *Tensor Geometry*, London: Pitman, 1979.

A very readable but technical text using linear (affine) algebra to study the local intrinsic geometry of spaces leading up to and including the geometry of the theory of relativity.

Gauss, C.F., *General Investigations of Curved Surfaces*, Hewlett, NY: Raven Press, 1965.

A translation into English of Gauss' early papers on surfaces.

Gray, A., *Modern Differential Geometry of Curves and Surfaces*, Akron: CRC, 1993.

This is a very extensive book based on computations using Mathematica©.

Henderson, David W., *Differential Geometry: A Geometric Introduction*, Upper Saddle River, NJ: Prentice Hall, 1998.

"In this book we will study a foundation for differential geometry based not on analytic formalisms but rather on these underlying geometric intuitions."

Koenderink, Jan J., *Solid Shape*, Cambridge: M.I.T. Press, 1990.

Written for engineers and applied mathematicians, this is a discussion of the extrinsic properties of three-dimensional shapes. There are connections with applications and a nice section called "Your way into the literature."

Kreyszig, Erwin, *Mathematical Expositions No. 11: Differential Geometry*, Toronto: University of Toronto Press, 1959.

"This book provides an introduction to the differential geometry of curves and surfaces in three-dimensional Euclidean space... In the theory of surfaces we make full use of the tensor calculus, which is developed as needed."

McCleary, John, *Geometry from a Differential Viewpoint*, Cambridge, UK: Cambridge University Press, 1994.

"The text serves as both an introduction to the classical differential geometry of curves and surfaces and as a history of ... the hyperbolic plane."

Morgan, Frank, *Riemannian Geometry: A Beginner's Guide*, Boston: Jones and Bartlett, 1993.

An accessible guide to Riemannian geometry including a chapter on the theory of relativity and the calculation of the precession in the orbit of Mercury.

Penrose, Roger, "The Geometry of the Universe," *Mathematics Today*, Lynn Steen, ed., New York: Springer-Verlag, 1978.

An expository discussion of the geometry of the universe.

Rovenski, V.Y., *Geometry of Curves and Surfaces with MAPLE*, Boston: Birkhäuser, 1998.

"This concise text on geometry with computer modeling presents some elementary methods for analytical modeling and visualization on curves and surfaces."

Santander, M., "The Chinese South-Seeking chariot: A simple mechanical device for visualizing curvature and parallel transport," *American Journal of Physics*, vol. 60, no. 9, pp. 782–87, 1992.

"An old mechanical device, the Chinese South-Seeking chariot, presumably designed to work on a flat plane, is shown to perform parallel transport on arbitrary surfaces. Its use affords experimental demonstration and even numerical checking (within a reasonable accuracy) of all the features of curvature and parallel transport of vectors in a two-dimensional surface."

Spivak, Michael, *A Comprehensive Introduction to Differential Geometry*, Wilmington, DE: Publish or Perish, 1979.

In five (!) volumes Spivak relates the subject back to the original sources. Volume V contains an extensive bibliography (to 1979).

Stahl, Saul, *The Poincaré Half-Plane*, Boston: Jones and Bartlett Publishers, 1993.

This text is an analytic introduction to some of the ideas of intrinsic differential geometry starting from the Calculus.

Thurston, William, *Three-Dimensional Geometry and Topology*, Vol. 1, Princeton, NJ: Princeton University Press, 1997.

This is a detailed excursion through the geometry and topology of 2- and 3-manifolds. "The style of exposition in this book is intended to encourage the reader to pause, to look around and to explore."

Weeks, Jeffrey, *The Shape of Space*, New York: Marcel Dekker, 1985.

An elementary but deep discussion of the geometry on different two- and three-dimensional spaces.

Di Dissections

Boltyanski (Boltianskii), Vladimir G., *Hilbert's Third Problem*, New York: John Wiley & Sons, 1978.

A discussion of dissections on the plane, sphere, and hyperbolic spaces.

Eves, Howard, *A Survey of Geometry,* Vol. 1, Boston: Allyn & Bacon, 1963.

A textbook that contains an extensive coverage of the dissection theory of polygons.

Frederickson, Greg, *Dissections*: *Plane and Fancy*, New York: Cambridge University Press, 1997.

This book is a collection of interesting dissection puzzles, old and new, and is an instructive manual on the art and science of geometric dissections.

Ho, Chung-Wu, "Decomposition of a Polygon into Triangles," *Mathematical Gazette*, vol. 60, pp.132–34, 1976.

This article contains a proof that all planar polygons can be dissected into triangles and discusses the many mistakes made by other (many well-known) authors in their "proofs" of the same result.

Sah, C.H., *Hilbert's Third Problem: Scissors Congruence*, London: Pitman, 1979.

A detailed discussion of the three-dimensional dissections.

DS Dimensions and Scale

Abbott, Edwin A., *Flatland*, New York: Dover Publications, Inc., 1952.

A fantasy about two-dimensional beings in a plane encountering the third dimension.

Banchoff, Thomas, and John Wermer, *Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimensions*, New York: Springer-Verlag, 1983.

"This book treats a number of themes that center on the notion of dimensions, tracing the different ways in which mathematicians and others have met them in their work."

Burger, Dionys, *Sphereland*, New York: Thomas Y. Crowell Co., 1965.

A sequel to Abbott's *Flatland*.

Kohl, Judith, and Herbert Kohl, *The View from the Oak: The Private Worlds of Other Creatures*, New York: Sierra Club Books/Charles Scribner's Sons, 1977.

This delightful book describes the various experiential worlds of different creatures and is a good illustration of intrinsic ways of thinking. Included are differing dimensions and scales of these worlds.

Morrison, Phillip, and Phylis Morrison, *Powers of Ten: About the Relative Size of Things in the Universe*, New York: Scientific American Books, Inc., 1982.

A beautiful book (and a video with the same title) that starts with a square meter on earth and then zooms out and in by powers of ten describing and illustrating at each power of ten what can be seen until it reaches (by zooming out) vast stretches of empty space in the universe or (by zooming in) the empty space within elementary particles.

Rucker, Rudy, *The Fourth Dimension*, Boston: Houghton Mifflin Co., 1984.

A history and description of various ways that people have considered the fourth dimension.

Rucker, Rudy, *Geometry, Relativity and the Fourth Dimension*, New York: Dover, 1977.

"[Author's] goal has been to present an intuitive picture of the curved space-time we call home."

GC Geometry in Different Cultures

Albarn, K., Jenny Miall Smith, Stanford Steele, Dinah Walker, *The Language of Pattern*, New York: Harper & Row, 1974.

An inquiry inspired by Islamic decoration.

Ascher, Marcia, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Pacific Grove, CA: Brooks/Cole, 1991.

A mostly anthropological look at the mathematics indigenous to several ancient cultures.

Bain, George, *Celtic Arts: The Methods of Construction*, London: Constable, 1977.

A description of the construction of Celtic patterns and designs.

Data, *The Science of the Sulba*, Calcutta: University of Calcutta, 1932.

A discussion of the mathematics in the *Sulbasutram* and traditional Hindi society.

Gerdes, Paulus, *Women, Art and Geometry in Southern Africa*, Trenton: Africa World Press, Inc., 1998.

"The main objective of the book *Women, Art and Geometry in Southern Africa* is to call attention to some mathematical aspects and ideas incorporated in the patterns invented by women in Southern Africa."

Gerdes, Paulus, *Geometry From Africa*: *Mathematical and Educational Explorations*, Washington: Mathematical Association of America, 1999.

"... we learn of the diversity, richness, and pleasure of mathematical ideas found in Sub-Saharan Africa. From a careful reading and working through this delightful book, one will find a fresh approach to mathematical inquiry as well as encounter a subtle challenge to Eurocentric discourses concerning the when, where, who, and why of mathematics."

Pinxten, R., Ingrid van Dooren, Frank Harvey, *The Anthropology of Space*, Philadelphia: University of Pennsylvania Press, 1983.

Concepts of geometry and space in the Navajo culture.

Zaslavsky, Claudia, *Africa Counts*, Boston: Prindle, Weber, and Schmidt, Inc., 1973.

A presentation of the mathematics in African cultures.

Hi History

Berggren, *Episodes in the Mathematics of Medieval Islam*, New York: Springer-Verlag, 1986.

Describes many examples that are difficult to find elsewhere of the mathematical contributions from medieval Islam.

Calinger, Ronald, *Classics of Mathematics*, Englewood Cliffs, NJ: Prentice Hall, 1995.

Mostly a collection of original sources in Western mathematics.

Carroll, Lewis, *Euclid and His Modern Rivals*, New York: Dover Publications, Inc., 1973.

Yes! Lewis Carroll of *Alice in Wonderland* fame was a geometer. This book is written as a drama; Carroll has Euclid defending himself against modern critics.

Critchlow, K., *Time Stands Still*, London: Gordon Fraser, 1979.

This book describes evidence of prehistoric scientific and mathematical knowledge.

Eves, Howard, *Great Moments in Mathematics* (*after 1650*), Dolciani Mathematical Expositions, vol. 7, Washington, DC: M.A.A., 1981.

This small book contains 20 lectures: 2 on non-Euclidean geometry and one on Klien's "Erlanger Program" which set out to delineate various aspects of mathematics (including especially geometry) according to which transformations left invarient important properties.

Fauvel, John, and Jeremy Gray, *The History of Mathematics: A Reader*, London: Macmillan Press, 1987.

"The selection of readings has been made for students of the Open University course MA290 *Topics in the History of Mathematics* ..."

Gray, Jeremy, *Ideas of Space*: *Euclidean, Non-Euclidean and Relativistic*, 2nd edition, Oxford: Oxford University Press, 1989.

A mostly historical account of Euclidean, non-Euclidean, and relativistic geometry.

Heath, T.L., *Mathematics in Aristotle,* Oxford: Clarendon Press, 1949.

Discusses the mathematical contributions of Aristotle.

Heath, T.L., ed., *Euclid*: *The Thirteen Books of the Elements*, New York: Dover, 1956.

This is the edition of Eulid's *Elements* to which one is usually referred. Heath has added a large collection of very useful historical and philosophical notes. His notes are more extensive than Euclid's text.

Heilbron, J.L., *Geometry Civilized: History, Culture, and Technique*, Oxford: Clarendon Press, 2000.

"For many centuries, geometry was part of high culture as well as an instrument of practical utility."

Joseph, George, *The Crest of the Peacock*, New York: I.B. Tauris, 1991.

A non-Eurocentric view of the history of mathematics.

Katz, Victor J., *A History of Mathematics: An Introduction*, Reading, MA: Addison-Wesley Longman, 1998.

"... designed for junior or senior mathematics majors who intend to teach in college or high school and thus concentrates on the history of those topics typically covered in an undergraduate curriculum or in elementary or high school."

Kline, Morris, *Mathematical Thought from Ancient to Modern Times*, Oxford: Oxford University Press, 1972.

A complete Eurocentric history of mathematical ideas including differential geometry (mostly the analytic side).

Laubenbacher, Reinhard, and David Pengelley, *Mathematical Expeditions: Chronicles by the Explorers*, New York: Springer, 1999.

Contains a 53-page chapter on "Geometry: The Parallel Postulate."

Newell, Virginia K., ed., *Black Mathematicians and Their Works*, Ardmore, PA: Dorrance, 1980.

A discussion of (mostly American) black mathematicians and their mathematics.

Richards, Joan, *Mathematical Visions*, Boston: Academic Press, 1988.

"The pursuit of geometry in Victorian England."

Rosenfeld, B.A., *A History of Non-Euclidean Geometry*, New York: Springer-Verlag, 1989.

An extensive history of non-Euclidean geometry based on original sources.

Seidenberg, A., "The Ritual Origin of Geometry," *Archive for the History of the Exact Sciences*, vol. 1, pp. 488–527, 1961.

In this article Seidenberg makes the case that much geometry originated from the needs of various religious rituals.

Snyder, John P., *Flattening the Earth*: *Two Thousand Years of Map Projections*, Chicago: University of Chicago Press, 1993.

A history and mathematical description of numerous map projections of the sphere.

Toth, I., "Non-Euclidean Geometry before Euclid," *Scientific American*, vol. 251, 1969.

Discusses the evidence of non-Euclidean geometry before Euclid.

Valens, Evans G., *The Number of Things: Pythagoras, Geometry and Humming Strings*, New York: E.P. Dutton and Company, 1964.

This is a book about ideas and is not a textbook. Valens leads the reader through dissections, golden mean, relations between geometry and music, conic sections, etc.

van der Waerden, B.L., *Science Awakening I: Egyptian, Babylonian, and Greek Mathematics*, Princeton Junction, NJ: The Scholar's Bookshelf, 1975.

"It is the intention to make this book scientific, but at the same time accessible to any one who has learned some mathematics in school and in college, and who is interested in the history of mathematics."

MP Models, Polyhedra

Barr, Stephen, *Experiments in Topology*, New York: Crowell, 1964.

Experimental topology that goes beyond the Möbius Band.

Cundy, M.H., and A.P. Rollett, *Mathematical Models*, Oxford: Clarendon, 1961.

Directions on how to make and understand various geometric models.

Ehrenfeucht, Aniela, *The Cube Made Interesting*, Waclaw Zawadowski, trans., New York: Pergamon Press, 1964.

"This book arose from popular scientific talks to teachers and school children." The discussion is illustrated with 3-D pictures using special glasses.

Lénárt, István, *Lénárt Sphere Construction Materials: Construction Materials for Another World of Geometry*, Berkeley: Key Curriculum Press, 1996.

This is a kit consisting of a transparent sphere, a spherical compass, and a spherical "straight edge" that doubles as a protractor. Other accessory materials are available.

Lyusternik, L.A., *Convex Figures and Polyhedra*, Boston: Heath, 1966.

A detailed but elementary study of convex figures.

Row, T. Sundra, *Geometric Exercises in Paper Folding*, New York: Dover, 1966.

How to produce various geometric constructions merely by folding a sheet of paper.

Senechal, Marjorie, and George Fleck, eds., *Shaping Space: A Polyhedral Approach*, Design Science Collection, Boston: Birkhauser, 1988.

This book is an accessible "exploration of the world of polyhedra, beginning with [an introduction] and concluding with an examination of the significance of polyhedral models in contemporary science and a survey of some recent advances and unsolved problems in mathematics."

Na Nature

Cook, T.A., *The Curves of Life*, New York: Dover Publications, 1979.

Subtitle: *Being an Account of Spiral Formations and their Applications to Growth in Nature, to Science, and to Art*.

Hildebrandt, Stefan, and Anthony Tromba, *Mathematics and Optimal Form*, New York: Scientific American Books, Inc., 1985.

"Combining striking photographs with a compelling text, authors ... give us a thoughtful account of the symmetry and regularity of nature's forms and patterns."

Kohl, Judith, and Herbert Kohl, *The View from the Oak: The Private Worlds of Other Creatures*, New York: Sierra Club Books/Charles Scribner's Sons, 1977.

This delightful books describes the various experiential worlds of different creatures and is a good illustration of intrinsic ways of thinking.

Mandelbrot, Benoit B., *The Fractal Geometry of Nature*, New York: W.H. Freeman and Company, 1983.

The book that started the popularity of fractal geometry.

McMahon, Thomas, and James Bonner, *On Size and Life*, New York: Scientific American Library, 1983.

A geometric discussion of the shapes and sizes of living things.

Thompson, D'Arcy, *On Growth and Form*, Cambridge: Cambridge University Press, 1961.

A classic on the geometry of the natural world.

NE Non-Euclidean Geometries (Mostly Hyperbolic)

Bonola, Roberto, *Non-Euclidean Geometry: A Critical and Historic Study of its Developments, and "The Theory of Parallels" by Nicholas Lobachevski with a supplement containing "The Science of Absolute Space" by John Bolyai*, New York: Dover, 1955.

"Bonola's Non-Euclidean Geometry is an elementary historical and critical study of the development of that subject."

N. V. Efimov, "Generation of singularities on surfaces of negative curvature [Russian]", *Mat. Sb.* (N.S.) 64 (106), pp. 286-320, 1964.

Efimov proves that it is impossible to have a C^2 isometric embedding of the hyperbolic plane onto a closed subset of Euclidean 3-space. These results are clarified for English-reading audiences in [**NE:** Milnor].

Greenberg, Marvin J., *Euclidean and Non-Euclidean Geometries: Development and History*, New York: Freeman, 1980.

This is a very readable textbook that includes some philosophical discussions.

Hilbert, David, "Über Flächen von konstanter gausscher Krümmung," *Transactions of the A.M.S.*, pp. 87–99, 1901.

Hilbert proves here that the hyperbolic plane does not have a real analytic (or C^4) isometric embedding onto a closed subset of Euclidean 3-space.

Kuiper, Nicolas, "On C^1-isometric embeddings, ii," *Nederl. Akad. Wetensch. Proc. Ser. A*, pp. 683–89, 1955.

Kuiper shows that there is a C^1 isometric embedding of the hyperbolic plane onto a closed subset of Euclidean 3-space.

Millman, Richard S., and George D. Parker, *Geometry: A Metric Approach with Models*, New York: Springer-Verlag, 1981.

A modern formal axiomatic approach.

Milnor, Tilla, Efimov's theorem about complete immersed surfaces of negative curvature, *Advances in Math.*, vol. 8, pp. 474–543, 1972.

Milnor clarifies for English readers the result in [NE: Efimov].

Petit, Jean-Pierre, *Euclid Rules OK? The Adventures of Archibald Higgins*, London: John Murray, 1982.

A pictorial, visual tour of non-Euclidean geometries.

Schwerdtfeger, Hans, *Geometry of Complex Numbers: Circle Geometry, Moebius Transformation, Non-Euclidean Geometry*, New York: Dover Publications, Inc., 1979.

This book uses complex numbers to analyze inversions in circles and then their relationship to hyperbolic geometry.

Singer, David A., *Geometry: Plane and Fancy*, New York: Springer, 1998.

"This book is about ... the idea of curvature and how it affects the assumptions about and principles of geometry."

Stahl, Saul, *The Poincaré Half-Plane*, Boston: Jones and Bartlett Publishers, 1993.

This text is an analytic introduction to some of the ideas of intrinsic differential geometry starting from the Calculus.

Trudeau, Richard J., *The Non-Euclidean Revolution*, Boston: Birkhäuser, 1987.

"Trudeau's book provides the reader with a non-technical description of the progress of thought from Plato and Euclid to Kant, Lobachevsky, and Hilbert."

Zage, "The Geometry of Binocular Visual Space," *Mathematics Magazine*, vol. 53, no. 5, pp. 289–94, 1980.

"... we relate the results of experiments in binocular vision to geometric models to arrive at the conclusion that the geometry of binocular visual space is [...] hyperbolic."

Ph Philosophy

Benacerraf, Paul, and Hilary Putman, *Philosophy of Mathematics: Selected Readings*, Cambridge: Cambridge University Press, 1964.

An interesting and useful selection of readings.

Lakatos, I., *Proofs and Refutations*, Cambridge: Cambridge University Press, 1976.

A deep but accessible book that uses an imaginary classroom dialogue in which the actual historical words of mathematicians are used to explore the evolving nature of mathematical ideas and to support the author's *quasi empirical* view of mathematics.

Rucker, Rudy, *Infinity and the Mind: The Science and Philosophy of the Infinite*, Boston: Birkhauser, 1982.

"This book discusses every kind of infinity: potential and actual, mathematical and physical, theological and mundane. Talking about infinity leads to many fascinating paradoxes. By closely examining these paradoxes we learn a great deal about the human mind, its powers, and its limitations."

Tymoczko, Thomas, *New Directions in the Philosophy of Mathematics*, Boston: Birkhauser, 1986.

An updated (to 1986) collection of readings.

RN Real Numbers

Epstein, Richard L., and Walter A. Carnielli, *Computability: Computable Functions, Logic, and the Foundations of Mathematics*, Pacific Grove, CA: Wadsworth & Brooks/Cole, 1989.

"This book... deals with a very basic problem: What is computable?"

Simpson, "The Infidel Is Innocent," *The Mathematical Intelligencer*, vol. 12, pp.42–51, 1990.

An accessible exposition of the nonstandard reals.

Turner, Peter R., "Will the 'Real' Real Arithmetic Please Stand Up?", *Notices of the A.M.S.*, vol. 38, pp.298–304, 1991.

An article about various finite representations of real numbers used in computing.

SE Surveys and General Expositions

Coxeter, H.S.M., and S.L. Greitzer, *Geometry Revisited*, New Mathematics Library 19, New York: The L.W. Singer Company, 1967.

"Using whatever means will best suit our purposes, let us revisit Euclid. Let us discover for ourselves a few of the newer results. Perhaps we may be able to recapture some of the wonder and awe that our first contact with geometry aroused."

Davis, P.J., and R. Hersh, *The Mathematical Experience*, Boston: Birkhauser, 1981.

A very readable collection of essays by two present-day mathematicians. I think every mathematics major should own this book.

Gorini, Catherine, ed., *Geometry at Work*, MAA Notes, vol. 53, Washington, DC: MAA, 2000.

A varied collection of writings on applications of geometry.

Hilbert, David, and S. Cohn-Vossen, *Geometry and the Imagination*, New York: Chelsea Publishing Co., 1983.

They state "it is our purpose to give a presentation of geometry, as it stands today [1932], in its visual, intuitive aspects." It includes an introduction to differential geometry, symmetry, and patterns (they call it "crystallographic groups"), and the geometry of spheres and other surfaces. Hilbert is the most famous mathematician of the first part of the 20th century.

SG Symmetry and Groups

Budden, F.J., *Fascination of Groups*, Cambridge: Cambridge University Press, 1972.

This is a fascinating book that relates algebra (groups) to geometry, music, and so forth, and has a nice description of symmetry and patterns.

Grünbaum, Branko, and G.C. Shepard, *Tilings and Patterns*, New York: W.H. Freeman, 1987.

A 700-page book detailing what is known about plane tilings and patterns.

Lyndon, Roger C., *Groups and Geometry*, New York: Cambridge University Press, 1985.

"This book is intended as an introduction, demanding a minimum of background, to some of the central ideas in the theory of groups and in geometry. It grew out of a course for advanced undergraduates and beginning graduate students."

Macgillavry, Caorline H., *Symmetry Aspects of M.C. Escher's Periodic Drawings*, Utrecht: Published for the International Union of Crystallography by A. Oosthoek's Uitgeversmaatschappij NV, 1965.

This volume describes a scheme for classifying periodic patterns with colors, using as examples Escher's drawings.

Martin, George E., *Transformation Geometry: An Introduction to Symmetry*, New York: Springer-Verlag, 1982.

"Our study of the automorphisms of the plane and of space is based on only the most elementary high-school geometry. In particular, group theory is *not* a prerequisite here. On the contrary, this modern approach to Euclidean geometry gives the concrete examples that are necessary to appreciate an introduction to group theory."

Montesinos, José María, *Classical Tessellations and Three-Manifolds*, New York: Springer Verlag, 1985.

"This book explores a relationship between classical tessellations and three-manifolds."

Robertson, Stewart A., *Polytopes and Symmetry*, New York: Cambridge University Press, 1985.

"These notes are intended to give a fairly systematic exposition of an approach to the symmetry classification of convex polytopes that casts some fresh light on classical ideas and generates a number of new theorems."

Weyl, Hermann, *Symmetry*, Princeton, NJ: Princeton University Press, 1952.

A readable discussion of all mathematical aspects of symmetry, especially its relation to art and nature — nice pictures. Weyl is a leading mathematician of this century.

Yale, Paul B., *Geometry and Symmetry*, New York: Dover, 1988.

"This book is an introduction to the geometry of Euclidean, affine and projective spaces with special emphasis on the important groups of symmetries of these spaces."

SP Spherical and Projective Geometry

Albert, A. Adrian, and Reuben Sandler, *An Introduction to Finite Projective Planes*, New York: Holt, Rinehart and Winston, 1968.

"In this book the authors have endeavored to introduce the subject of finite projective planes as it has developed during the last twenty years."

Coxeter, H.S.M., *The Real Projective Plane*, New York: Cambridge University Press, 1955.

"This introduction to projective geometry can be understood by anyone familiar with high-school geometry and algebra. The restriction to real geometry of two dimensions makes it possible for every theorem to be illustrated by a diagram."

Lénárt, István, *Non-Euclidean Adventures on the Lénárt Sphere: Activities comparing planar and spherical geometry*, Berkeley: Key Curriculum Press, 1996.

This is a manual for high school teachers using the Lénárt Sphere.

Todhunter, Isaac, *Spherical Trigonometry*, London: Macmillan, 1886.

All you want to know, and more, about trigonometry on the sphere. Well-written with nice discussions of surveying.

TG Teaching Geometry

Malkevitch, Joseph, *Geometry's Future second edition*, USA: COMAP, 1991.

Proceedings of a COMAP conference "of a small group of geometers to study what could be done to revitalize geometry in our colleges, and what effects this might have on the teaching of geometry in general."

Mammana, Carnelo, and Vinicio Villani, ed., *Perspectives on the Teaching of Geometry for the 21st Century: ICMI Study*, Dortrecht, Netherlands: Kluwer, 1998.

Contains papers and the edited summaries and conclusions from the ICMI Study.

Traylor, Reginald, *Creative Teaching: Heritage of R.L. Moore*, Houston: University of Houston, 1972. Also at http://at.yorku.ca/i/a/a/b/21.dir/

A mathematical teaching biography of R.L. Moore and list of his mathematical descendents.

Zimmermann, Walter and Cunningham, Steve, *Visualization in Teaching and Learning Mathematics*, USA: Mathematical Association of America, 1991.

"A project sponsored by the Committee on Computers in Mathematics Education of the M.A.A."

Tp Topology

Arnold, B.H., *Intuitive Concepts in Elementary Topology*, Englewood Cliffs: Prentice-Hall, Inc., 1962.

"Topology is presented here from the intuitive, rather than the axiomatic viewpoint. Some concepts are introduced, discussed and used informally, on the basis of the student's experience."

Blackett, Donald W., *Elementary Topology*, New York: Academic Press, 1967.

Contains a combinatorial-based proof of the classification of 2-manifolds.

Cairns, Stewart Scott, *Introductory Topology*, New York: The Ronald Press Company, 1961.

"This book is the culmination of repeatedly revised sets of class notes used by the author in teaching introductory topology courses. Its purpose is to progress as far as practicable into the fundamental concepts and the principal results of homology theory, both in their combinatorial development and in their application to topological spaces."

Francis, G.K., *A Topological Picturebook*, New York: Springer Verlag, 1987.

Francis presents elaborate and illustrative drawings of surfaces and provides guidelines for those who wish to produce such drawings.

Francis, G.K., and Jeffrey R. Weeks, "Conway's ZIP Proof", *American Mathematical Monthly*, vol. 106, May 1999, pages 393–99, May 1999.

A new proof of the classification of (triangulated) surfaces (2-manifolds).

Tx Geometry Texts

Berger, M., *Geometry I & II*, Sole, M. and Levy, trans., New York: Springer-Verlag, 1987.

"This two-volume textbook is the translation of the French book 'Géométrie' originally published in five volumes. It gives a detailed treatment of classical geometry and provides a comprehensive and unified reference source for all the subfields of geometry, including crystallographic groups; affine, Euclidean, Spherical and hyperbolic geometries; projective geometry; geometry of triangles, tetrahedra, circles and spheres; convex sets, convex and regular polyhedra, etc."

Bruni, James V., *Experiencing Geometry*, Wadsworth Publishing Company, Inc., Belmont, CA, 1977.

"This book is meant to be an informal, intuitive *introduction* to geometry. If you glance through the book, you will find none of the formal theorems of proofs you might expect in a mathematics textbook. You will find numerous concrete models and illustrations intended to be a springboard for the discovery of geometric ideas. Through guided observation, experimentation, and the use of your intuition, you will investigate some fundamental geometric concepts. These experiences can serve as preparation for a more formal, abstract, and logically precise study of different kinds of geometry."

Coxeter, H.S.M., *Introduction to Geometry*, New York: Wiley, 1969.

This is a collection of diverse topics including non-Euclidean geometry, symmetry, patterns, and much, much more. Coxeter is one of the foremost living geometers.

Hansen, Vagn Lundsgaard, *Shadows of the circle: Conic Sections, Optimal Figures and Non-Euclidean Geometry*, Singapore: World Scientific, 1998.

"It is my hope that these topics will be an inspiration in connection with teaching of geometry at various levels including upper secondary school and college education."

Jacobs, Harold R., *Geometry*, San Fransisco: W.H. Freeman and Co., 1974.

A high-school-level text based on guided discovery.

Martin, George E., *Geometric Constructions*, New York: Springer, 1998.

A geometry textbook based on ruler and compass constructions.

Pedoe, Dan, *Geometry: A Comprehensive Course*, New York: Dover Publications, Inc., 1970.

"The main purpose of the course was to increase geometrical, and therefore mathematical understanding, and to help students to enjoy geometry. This is also the purpose of my book."

Serra, Michael, *Discovering Geometry: An Inductive Approach*, Berkeley, CA: Key Curriculum Press, 1989.

A high school text that promotes discovery.

Shurman, Jerry, *Geometry of the Quintic*, New York: John Wiley & Sons, Inc., 1997.

An advanced undergraduate text that uses the icosahedron to solve quintic equations. Along the way, he explores the Riemann sphere, group representations, and invariant functions.

Sibley, Thomas Q., *The Geometric Viewpoint: A Survey of Geometries*, Reading, MA: Addison Wesley, 1998.

"Geometry combines visual delights and powerful abstractions, concrete intuitions and general theories, historical perspective and contemporary applications, and surprising insights and satisfying certainty. In this textbook, I try to weave together these facets of geometry. I also want to convey the multiple connections that different topics in geometry have with each other and that geometry has with other areas of mathematics."

Singer, David A., *Geometry: Plane and Fancy*, New York: Springer, 1998.

"This book is about... the idea of curvature and how it affects the assumptions about and principles of geometry."

Wallace, Edward C. and Stephen F. West, *Roads to Geometry*, Upper Saddle River, NJ: Prentice Hall, Inc., 1998.

"The goal of this book is to provide a geometric experience which clarifies, extends, and unifies concepts which are generally discussed in traditional high school geometry courses and to present additional topics which assist in gaining a better understanding of elementary geometry."

Un The Physical Universe

Ferris, Timothy, *The Whole Shebang*: *A State-of-the-Universe(s) Report*, New York: Simon & Schuster, 1997.

"This book aims to summarize the picture of the universe that science has adduced..., and to forecast an exciting if unsettling new picture that may emerge in the near future."

Guth, Alan H., *The Inflationary Universe*: *The Quest for a New Theory of Cosmic Origins*, Reading, MA: Perseus Books, 1997.

"The inflationary universe is a theory of the 'bang' of the big bang."

Osserman, Robert, *Poetry of the Universe*: *A Mathematical Exploration of the Cosmos*, New York: Anchor Books, 1995.

"What is the shape of the universe, and what do we mean by the curvature of space? One aim of this book is to make absolutely clear and understandable both the meanings of those questions and the answers to them. Little or no mathematical background is needed..."

Rees, Martin, *Before the Beginning*: *Our Universe and Others*, Reading, MA: Perseus Books, 1997.

"This book presents an individual view on cosmology — how we perceive our universe, what the current debates are about, and the scope and limits of our future knowledge."

Z Miscellaneous

Davis, Phillip, *The Thread*: *A Mathematical Yarn*, Boston: Birkhäuser, 1983.

Contains a story about the discovery of the mechanism that turns circular motion into straight line motion and thus can be used for drawing a straight line.

Kempe, A.B., *How to Draw a Straight Line*, London: Macmillan, 1877.

This small book contains a discussion and description of numerous curve-drawing devices including ones that will draw straight lines.

Snyder, John P., *Flattening the Earth*: *Two Thousand Years of Map Projections*, Chicago: University of Chicago Press, 1993.

A history and mathematical description of numerous map projections of the sphere.

Sobel, Dava, *Longitude*: *The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time*, New York: Penguin Books, 1995.

An account of the struggles to develop a method for determining the longitude of ships at sea.