propositions having a negative copula results from laws already known, especially from the formulas of De Morgan and the law of contraposition. We shall enumerate the chief formulas derived from it.

The principle of composition gives rise to the following formulas:

$$(c \nless ab) = (c \nless a) + (c \nless b),$$
$$(a + b \nless c) = (a \nless c) + (b \nless c),$$

whence come the particular instances

$$(ab \neq 1) = (a \neq 1) + (b \neq 1),$$
$$(\mathrm{a} + b \neq 0) = (a \neq 0) + (c \neq 0).$$

From these may be deduced the following important implications:

$$(a \neq 0) < (a + b \neq 0),$$
$$(a \neq 1) < (ab \neq 1).$$

From the principle of the syllogism, we deduce, by the law of transposition,

$$(a < b)\ (a \neq 0) < (b \neq 0),$$
$$(a < b)\ (b \neq 1) < (a \neq 1).$$

The formulas for transforming inclusions and equalities give corresponding formulas for the transformation of non-inclusions and inequalities,

$$(a \nless b) = (ab' \neq 0) \qquad = (a' + b \neq 1),$$
$$(a \neq b) = (ab' + a'b \neq 0) \quad = (ab + a'b' \neq 1).$$

54. Solution of an Inequation with One Unknown.— If we consider the conditional inequality (*inequation*) with one unknown

$$ax + bx' \neq 0,$$

we know that its first member is contained in the sum of its coefficients

$$ax + bx' < a + b.$$

From this we conclude that, if this inequation is verified, we have the inequality

$$a + b \neq 0.$$

This is the necessary condition of the solvability of the inequation, and the resultant of the elimination of the unknown x. For, since we have the equivalence

$$\prod_x (ax + bx' = 0) = (a + b = 0),$$

we have also by contraposition the equivalence

$$\sum_x (ax + bx' \neq 0) = (a + b \neq 0).$$

Likewise, from the equivalence

$$\sum_x (ax + bx' = 0) = (ab = 0),$$

we can deduce the equivalence

$$\prod_x (ax + bx' \neq 0) = (ab \neq 0),$$

which signifies that the necessary and sufficient condition for the inequation to be always true is

$$(ab \neq 0);$$

and, indeed, we know that in this case the equation

$$(ax + bx' = 0)$$

is impossible (never true).

Since, moreover, we have the equivalence

$$(ax + bx' = 0) = (x = a'x + bx'),$$

we have also the equivalence

$$(ax + bx' \neq 0) = (x \neq a'x + bx').$$

Notice the significance of this solution:

$$(ax + bx' \neq 0) = (ax \neq 0) + (bx' \neq 0) = (x \not< a') + (b \not< x).$$

"Either x is not contained in a', or it does not contain b". This is the negative of the double inclusion

$$b < x < a'.$$

Just as the product of several equalities is reduced to one single equality, the sum (the alternative) of several inequalities may be reduced to a single inequality. But neither several alternative equalities nor several simultaneous inequalities can be reduced to one.

55. System of an Equation and an Inequation.—We shall limit our study to the case of a simultaneous equality and inequality. For instance, let the two premises be

$$(ax + bx' = 0)\ (cx + dx' \neq 0).$$

To satisfy the former (the equation) its resultant $ab = 0$ must be verified. The solution of this equation is

$$x = a'x + bx'.$$

Substituting this expression (which is equivalent to the equation) in the inequation, the latter becomes

$$(a'c + ad)x + (bc + b'd)x' \neq 0.$$

Its resultant (the condition of its solvability) is

$$(a'c + ad + bc + b'd \neq 0) = [(a' + b)c + (a + b')d \neq 0],$$

which, taking into account the resultant of the equality,

$$(ab = 0) = (a' + b = a') = (a + b' = b')$$

may be reduced to

$$a'c + b'd \neq 0.$$

The same result may be reached by observing that the equality is equivalent to the two inclusions

$$(x < a')\ (x' < b'),$$

and by multiplying both members of each by the same term

$$(cx < a'c)\ (dx' < b'd) < (cx + dx' < a'c + b'd)$$

$$(cx + dx' \neq 0) < (a'c + b'd \neq 0).$$

This resultant implies the resultant of the inequality taken alone

$$c + d \neq 0,$$

so that we do not need to take the latter into account. It