cepts and the calculus of propositions become reduced to but one, the calculus of classes, or, as LEIBNIZ called it, the theory of the whole and part, of that which contains and that which is contained. But as a matter of fact, the calculus of concepts and the calculus of propositions present certain differences, as we shall see, which prevent their complete identification from the formal point of view and consequently their reduction to a single "calculus of classes".

Accordingly we have in reality three distinct calculi, or, in the part common to all, three different interpretations of the same calculus. In any case the reader must not forget that the logical value and the deductive sequence of the formulas does not in the least depend upon the interpretations which may be given them, and, in order to make this necessary abstraction easier, we shall take care to place the symbols "C. I." (*conceptual interpretation*) and "P. I." (*propositional interpretation*) before all interpretative phrases. These interpretations shall serve only to render the formulas intelligible, to give them clearness and to make their meaning at once obvious, but never to justify them. They may be omitted without destroying the logical rigidity of the system.

In order not to favor either interpretation we shall say that the letters represent *terms;* these terms may be either concepts or propositions according to the case in hand. Hence we use the word *term* only in the logical sense. When we wish to designate the "terms" of a sum we shall use the word *summand* in order that the logical and mathematical meanings of the word may not be confused. A term may therefore be either a factor or a summand.

3. Relation of Inclusion.—Like all deductive theories, the algebra of logic may be established on various systems of principles[1]; we shall choose the one which most nearly

[1] See HUNTINGTON, "Sets of Independent Postulates for the Algebra of Logic", *Transactions of the Am. Math. Soc.*, Vol. V, 1904, pp. 288—309. [Here he says: "Any set of consistent postulates would give rise to a corresponding algebra, viz., the totality of propositions which follow

approaches the exposition of SCHRÖDER and current logical interpretation.

The fundamental relation of this calculus is the binary (two-termed) relation which is called *inclusion* (for classes), *subsumption* (for concepts), or *implication* (for propositions). We will adopt the first name as affecting alike the two logical interpretations, and we will represent this relation by the sign $<$ because it has formal properties analogous to those of the mathematical relation $<$ ("less than") or more exactly $\leq$, especially the relation of not being symmetrical. Because of this analogy SCHRÖDER represents this relation by the sign $\Subset$ which we shall not employ because it is complex, whereas the relation of inclusion is a simple one.

In the system of principles which we shall adopt, this relation is taken as a primitive idea and is consequently indefinable. The explanations which follow are not given for the purpose of *defining* it but only to indicate its meaning according to each of the two interpretations.

C. I.: When a and b denote concepts, the relation $a < b$ signifies that the concept a is subsumed under the concept b; that is, it is a species with respect to the genus b. From the extensive point of view, it denotes that the class of a's is contained in the class of b's or makes a part of it; or, more concisely, that "All a's are b's". From the comprehensive point of view it means that the concept b is contained in the concept a or makes a part of it, so that consequently the character a implies or involves the character b. Example: "All men are mortal"; "Man implies mortal"; "Who says man says mortal"; or, simply, "Man, therefore mortal".

P. I.: When a and b denote propositions, the relation $a < b$ signifies that the proposition a implies or involves the proposition b, which is often expressed by the hypothetical judgment, "If a is true, b is true"; or by "a implies b"; or more simply by "a, therefore b". We see that in both inter-

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from these postulates by logical deductions. Every set of postulates should be free from redundances, in other words, the postulates of each set should be *independent*, no one of them deducible from the rest."]

pretations the relation $<$ may be translated approximately by "therefore".

Remark.—Such a relation as "$a < b$" is a proposition, whatever may be the interpretation of the terms a and b. Consequently, whenever a $<$ relation has two like relations (or even only one) for its members, it can receive only the propositional interpretation, that is to say, it can only denote an implication.

A relation whose members are simple terms (letters) is called a *primary* proposition; a relation whose members are primary propositions is called a *secondary* proposition, and so on.

From this it may be seen at once that the propositional interpretation is more homogeneous than the conceptual, since it alone makes it possible to give the same meaning to the copula $<$ in both primary and secondary propositions.

4. Definition of Equality.—There is a second copula that may be defined by means of the first; this is the copula $=$ ("equal to"). By definition we have

$$a = b,$$

whenever

$$a < b \text{ and } b < a$$

are true at the same time, and then only. In other words, the single relation $a = b$ is equivalent to the two simultaneous relations $a < b$ and $b < a$.

In both interpretations the meaning of the copula $=$ is determined by its formal definition:

C. I.: $a = b$ means, "All a's are b's and all b's are a's"; in other words, that the classes a and b coincide, that they are identical.[1]

P. I.: $a = b$ means that a implies b and b implies a; in

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[1] This does not mean that the concepts a and b have the same meaning. Examples: "triangle" and "trilateral", "equiangular triangle" and "equilateral triangle".