1. Introduction.—The algebra of logic was founded by George Boole (1815—1864); it was developed and perfected by Ernst Schröder (1841—1902). The fundamental laws of this calculus were devised to express the principles of reasoning, the "laws of thought". But this calculus may be considered from the purely formal point of view, which is that of mathematics, as an algebra based upon certain principles arbitrarily laid down. It belongs to the realm of philosophy to decide whether, and in what measure, this calculus corresponds to the actual operations of the mind, and is adapted to translate or even to replace argument; we cannot discuss this point here. The formal value of this calculus and its interest for the mathematician are absolutely independent of the interpretation given it and of the application which can be made of it to logical problems. In short, we shall discuss it not as logic but as algebra.

2. The Two Interpretations of the Logical Calculus.—There is one circumstance of particular interest, namely, that the algebra in question, like logic, is susceptible of two distinct interpretations, the parallelism between them being almost perfect, according as the letters represent concepts or propositions. Doubtless we can, with Boole and Schröder, reduce the two interpretations to one, by considering the concepts on the one hand and the propositions on the other as corresponding to *assemblages* or *classes*; since a concept determines the class of objects to which it is applied (and which in logic is called its *extension*), and a proposition determines the class of the instances or moments of time in which it is true (and which by analogy can also be called its extension). Accordingly the calculus of concepts and the calculus of propositions become reduced to but one, the calculus of classes, or, as LEIBNIZ called it, the theory of the whole and part, of that which contains and that which is contained. But as a matter of fact, the calculus of concepts and the calculus of propositions present certain differences, as we shall see, which prevent their complete identification from the formal point of view and consequently their reduction to a single "calculus of classes".

Accordingly we have in reality three distinct calculi, or, in the part common to all, three different interpretations of the same calculus. In any case the reader must not forget that the logical value and the deductive sequence of the formulas does not in the least depend upon the interpretations which may be given them, and, in order to make this necessary abstraction easier, we shall take care to place the symbols "C. I." (*conceptual interpretation*) and "P. I." (*propositional interpretation*) before all interpretative phrases. These interpretations shall serve only to render the formulas intelligible, to give them clearness and to make their meaning at once obvious, but never to justify them. They may be omitted without destroying the logical rigidity of the system.

In order not to favor either interpretation we shall say that the letters represent *terms;* these terms may be either concepts or propositions according to the case in hand. Hence we use the word *term* only in the logical sense. When we wish to designate the "terms" of a sum we shall use the word *summand* in order that the logical and mathematical meanings of the word may not be confused. A term may therefore be either a factor or a summand.

3. Relation of Inclusion.—Like all deductive theories, the algebra of logic may be established on various systems of principles[1]; we shall choose the one which most nearly

[1] See HUNTINGTON, "Sets of Independent Postulates for the Algebra of Logic", *Transactions of the Am. Math. Soc.*, Vol. V, 1904, pp. 288—309. [Here he says: "Any set of consistent postulates would give rise to a corresponding algebra, viz., the totality of propositions which follow