Advanced Studies in Pure Mathematics 32, 2001
Groups and Combinatorics — in memory of Michio Suzuki
pp. 337–346

A Remark on the Loewy Structure for the Three Dimensional Projective Special Unitary Groups in Characteristic 3

Shigeo Koshitani[1] and Naoko Kunugi

§1. Introduction and Notation

The purpose of this note is to give an alternative and easier proof of a recent result by K. Hicks [6, Theorem 1.1], which was on the Loewy and socle structure of the projective indecomposable modules in the principal 3-block of the projective special unitary group $\mathrm{PSU}_3(q^2) = \mathrm{U}_3(q)$ for a power q of a prime satisfying $q \equiv 2$ or $5 \pmod 9$ over an algebraically closed field of characteristic 3. In her paper K. Hicks used so-called Auslander-Reiten theory on representations of artin algebras (see [1]). Actually, in her paper [6], the key tool was a result, which was due to K. Erdmann [4] and S. Kawata [8] on Auslander-Reiten quivers of type A_∞ for group algebras of finite groups. On the other hand, our proof does not need the Auslander-Reiten theory (except a result due to P. Webb [15]) but just well-known results on modular representation theory of finite groups.

We use the following notation and terminology. Throughout this paper, k is always an algebraically closed field of characterictic $p > 0$, and G is always a finite group. For an element $g \in G$ we denote by $|g|$ the order of g. For a power q of a prime, $\mathbb{F}_q$ is the field of q elements, and we use the notation $\mathrm{GL}_n(q)$, $\mathrm{SL}_n(q)$, $\mathrm{PGL}_n(q)$, $\mathrm{PGU}_n(q)$, $\mathrm{PSU}_n(q)$ for a positive integer n in a standard fashion (see [7]). We denote by C_n the cyclic group of order n for a positive integer n. Let A be a finite-dimensional k-algebra. Then, $A^\times$ denotes the set of all units (invertible elements) in A, and $J(A)$ denotes the Jacobson radical of A. In this paper *modules* mean always finitely generated right modules, unless stated otherwise. Let M be an A-module. We denote by $\mathrm{Soc}(M)$ and $P(M)$ the socle of M and the projective cover of M, respectively.

---

[1] This work was partially supported by the JSPS (Japan Society for Promotion of Science).

Received March 3, 1999.

Let $J = J(kG)$. Then, we write $j(M)$ for the Loewy length of M, that is, $j(M)$ is the least positive integer j such that $M{\cdot}J^j = 0$. Then, for each $i = 1, \cdots, j(M)$, we can define the i-th Loewy layer $L_i(M)$ and i-th socle $\mathrm{Soc}_i(M)$ of M, namely, $L_i(M) = M{\cdot}J^{i-1}/M{\cdot}J^i$ and the i-th socle of M is defined inductively by $\mathrm{Soc}_0(M) = M$ and $\mathrm{Soc}_i(M)/\mathrm{Soc}_{i-1}(M) = \mathrm{Soc}(M/\mathrm{Soc}_{i-1}(M))$ for $i = 1, 2, \cdots, j(M)$. Let $M^* = \mathrm{Hom}_k(M, k)$ be the dual of M, which can be considered as a right kG-module as well via $(\phi{\cdot}g)(m) = \phi(mg^{-1})$ for any $m \in M$, $g \in G$ and $\phi \in \mathrm{Hom}_k(M, k)$. Then, M^* is called the (k-)*dual* of M. We say that M is *self-dual* if $M \cong M^*$ as right kG-modules.

From now on, let assume that A is a block ideal of the group algebra kG. Then, we write $\mathrm{Irr}(A)$ and $\mathrm{IBr}(A)$ respectively for the set of all irreducible ordinary characters of G in A and the set of all irreducible Brauer characters of G in A (note that sometimes we mean by $\mathrm{IBr}(A)$ the set of all non-isomorphic simple kG-modules in A). We write $k(A)$ and $\ell(A)$ respectively for the numbers of all elements in the sets $\mathrm{Irr}(A)$ and $\mathrm{IBr}(A)$. For simple kG-modules S and T, $c(S, T) = c_{S,T}$ denotes the Cartan invariant with respect to S and T. We denote by k_G the trivial kG-module. For other notation and terminology we follow the books of Landrock [12] and Nagao-Tsushima [13].

§1. $\mathbf{PSU}_3(q^2)$

In this section we give some remarks on $\mathrm{PSU}_3(q^2)$. First of all, we can define the 3-dimensional special unitary group $\mathrm{SU}_3(q^2)$ over the finite field $\mathbb{F}_{q^2}$ of q^2 elements for a power q of a prime such that

$$\mathrm{SU}_3(q^2) \;=\; \{X \in \mathrm{SL}_3(q^2) \mid X{\cdot}{}^t\overline{X} = I_3\}$$

where I_3 is the unit matrix of size 3×3, ${}^tY$ is the transposed matrix of a matrix Y and $\overline{Y}$ is the image of a matrix Y by the Frobenius map $\mathbb{F}_{q^2} \to \mathbb{F}_{q^2}$ with $\alpha \mapsto \alpha^q$, namely, $\overline{Y} = ({y_{ij}}^q)_{i,j}$ if $Y = (y_{ij})_{i,j}$ and $y_{ij} \in \mathbb{F}_{q^2}$, since there exists a normal orthogonal basis with respect to f, where f is a non-degenerate Hermite form over a 3-dimensional $\mathbb{F}_{q^2}$-vector space which defines $\mathrm{SU}_3(q^2)$ (see [7, II 10.4 Satz]). Throughout this paper, we assume that a power q of a prime satisfies a condition

$$\textbf{(2.1)} \qquad q \equiv 2 \text{ or } 5 \pmod 9.$$

Since the multiplicative group ${\mathbb{F}_{q^2}}^\times$ is a cyclic group of order $q^2 - 1$, let σ be a generator of it, namely, ${\mathbb{F}_{q^2}}^\times = \langle\sigma\rangle$ and we fix σ. Then, let

$\omega = \sigma^{(q^2-1)/3}$ and we fix ω (note that $q^2 - 1$ is divisible by 3 from (2.1)). Now, we can define

$$\textbf{(2.2)} \qquad G = \mathrm{PSU}_3(q^2) = \mathrm{SU}_3(q^2)/Z$$

where Z is the center of $\mathrm{SU}_3(q^2)$ and $Z = \{\omega^i \cdot I_3 \in \mathrm{SL}_3(q^2) \mid i = 0, 1, 2\}$ so that $Z \cong C_3$. Throughout this paper we write elements of G and $\mathrm{PGL}_3(q^2)$ just in forms of (3×3)-matrices. Let

$$\textbf{(2.3)} \qquad \beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \in \mathrm{PGL}_3(q^2).$$

Then, $\beta \in \widetilde{G} - G$ and $|\beta| = 3$ where $\widetilde{G} = \mathrm{PGU}_3(q^2)$. As in [14], let

$$\textbf{(2.4)} \qquad st' = (q-1)(q^2 - q + 1)/3.$$

Notation. In the rest of this paper, we assume that k is an algebraically closed field of characteristic 3 and that q is a power of a prime satisfying (2.1), and we use the notation G, $\widetilde{G}$, β and st' as in (2.2)–(2.4).

§2. Decomposition matrix and Cartan matrix for G

In this section we list the decomposition matrix and the Cartan matrix for G for a prime 3. Here we use the notation k, G, $\widetilde{G}$, β and st' as in §2. We denote by A the principal block of kG.

(3.1) Lemma. (i) *The decomposition matrix and the Cartan ma-*

trix of the principal block A of G for a prime 3 *are*

	$S(0)$	$S(1)$	$S(2)$	$S(3)$	S
χ_1	1	.	.	.	.
$\chi_{st'}^{(1)}$	.	1	.	.	.
$\chi_{st'}^{(2)}$	.	.	1	.	.
$\chi_{st'}^{(3)}$	.	.	.	1	.
χ_{q^2-q}	.	.	.	.	1
χ_{q^3}	1	1	1	1	2

	$P(0)$	$P(1)$	$P(2)$	$P(3)$	$P(S)$
$S(0)$	2	1	1	1	2
$S(1)$	1	2	1	1	2
$S(2)$	1	1	2	1	2
$S(3)$	1	1	1	2	2
S	2	2	2	2	5

where $S(0) = k_G$, *the subindices of* χ*'s above mean the degrees,* $S(0)$, $S(1)$, $S(2)$, $S(3)$ *and* S *are all simple* kG*-modules in* A, *and* $P(i) = P(S(i))$ *for* $i = 0, 1, 2, 3$.

(ii) *All simple* kG*-modules in* A *are self-dual. and the element* $\beta \in \widetilde{G}$ *of order* 3 *acts on* $\mathrm{Irr}(A) = \{\chi_1, \chi_{st'}^{(1)}, \chi_{st'}^{(2)}, \chi_{st'}^{(3)}, \chi_{q^2-q}, \chi_{q^3}\}$ *such that*

$$\chi_1{}^{\beta} = \chi_1,$$

$$(\chi_{st'}^{(1)})^{\beta} = \chi_{st'}^{(2)}, \quad (\chi_{st'}^{(2)})^{\beta} = \chi_{st'}^{(3)}, \quad (\chi_{st'}^{(3)})^{\beta} = \chi_{st'}^{(1)},$$

$$(\chi_{q^2-q})^{\beta} = \chi_{q^2-q}, \quad (\chi_{q^3})^{\beta} = \chi_{q^3}.$$

Proof. (i) The assertion is obtained by the result of Geck [5, pp.571–573, Theorem 4.5], and a standard argument (see [3, Lemmas 66.1 and 64.3(1)]).

(ii) We get the self-dualities by (3.1), (i) and [5, Table 3.1, p.569]. It follows from [14, Table 2, p.492], [5, p.569, p.571] and [9, Tafel I, p.141] that $(\chi_{st'}^{(i)})^{\beta} = \chi_{st'}^{(i+1)}$ for $i = 0, 1, 2$, where the index i is considered modulo 3. The rest in (ii) is easy. Q.E.D.

Notation. In the rest of this paper, we use the notation χ_i, $\chi_i^{(j)}$, k_G, $S(i)$, S as in (3.1).

§3. Projectives in the principal 3-block of G

In this section we investigate the Loewy and socle series of projective indecomposable kG-modules in the principal block A of kG. We use the notation $S(0) = k_G$, $S(1)$, $S(2)$, $S(3)$ and S which means all non-isomorphic simple kG-modules in the principal block A of kG as in (3.1).

(4.1) Theorem. *The Loewy and socle series of the projective indecomposable kG-modules are*

$$P(S(i)) = \begin{array}{ccc} & S(i) & \\ & S & \\ S(j) & S(k') & S(\ell) \\ & S & \\ & S(i) & \end{array} \qquad P(S) = \begin{array}{cccc} & & S & \\ S(0) & S(1) & S(2) & S(3) \\ & S & S & S \\ S(0) & S(1) & S(2) & S(3) \\ & & S & \end{array}$$

where $\{i, j, k', \ell\} = \{0, 1, 2, 3\}$ *and* $S(0) = k_G$.

Proof. Let $J = J(kG)$ and $A = B_0(kG)$, the principal block of kG. Let $S(0) = k_G$, $S(4) = S$ and $P(i) = P(S(i))$ for each $i = 0, 1, 2, 3, 4$. We write $c(i, j)$ for $c(S(i), S(j))$ for each i, j. By (3.1)(i), we know that $k(A) - \ell(A) = 1$. Hence it follows from a result of Brandt [2, Theorem B] that

$$(0) \qquad \mathrm{Ext}^1_{kG}(S(i), S(i)) = 0 \qquad \text{for all } i = 0,\ 1,\ 2,\ 3,\ 4.$$

We get from (3.1)(ii) that $S(0)$ and $S(1)$ are both self-dual and that $c(0, 1) = 1$. Hence, if $\mathrm{Ext}^1_{kG}(S(0), S(1)) \neq 0$, then the self-duality implies that $S(1)$ is a direct summand of the heart $H(P(0)) = P(0){\cdot}J/\mathrm{Soc}(P(0))$ of $P(0)$, which means that $H(P(0))$ is decomposable by the Cartan matrix in (3.1)(i), contradicting a result of Webb [15, Theorem E].

Therefore, $\mathrm{Ext}^1_{kG}(S(0), S(1)) = 0$. Hence, by using the automorphism β of kG in (3.1)(ii), we have $\mathrm{Ext}^1_{kG}(S(0), S(i)) = 0$ for all $i = 1, 2, 3$.

Similarly, if we assume that $\dim_k[\mathrm{Ext}^1_{kG}(S(0), S(4))] = 2$, then it follows from the self-duality and the Cartan matrix for A in (3.1)(i) that the heart $H(P(0))$ is decomposable, contradicting [15, Theorem E].

Therefore, the self-duality says that $P(0)/P(0){\cdot}J^2$ and $\mathrm{Soc}_2(P(0))$ are both uniserial with

$$L_2(P(0)) \cong S(4) \cong \mathrm{Soc}_2(P(0))/\mathrm{Soc}_1(P(0)).$$

Hence, by the Cartan matrix in (3.1)(i), there left only $S(1)$, $S(2)$, $S(3)$ with multiplicity one in the composition factors of $P(0)$, respectively, whose positions in the Loewy series of $P(0)$ are not determined. So, the automorphism β in (3.1)(ii) implies that $S(1) \bigoplus S(2) \bigoplus S(3) \hookrightarrow L_3(P(0))$, completing the Loewy structure of $P(0)$. Hence, by the self-dualities, we get that the Loewy and socle series of $P(0)$ has the form

$$(1) \qquad P(0) \;=\; \begin{array}{ccc} & S(0) & \\ & S(4) & \\ S(1) & S(2) & S(3). \\ & S(4) & \\ & S(0) & \end{array}$$

Now, it follows from a result of Landrock [11, Theorem E] and (1) that $S(0) \hookrightarrow L_3(P(i))$ for all $i = 1, 2, 3$, $S(0) \hookrightarrow L_2(P(4))$ and $S(0) \hookrightarrow L_4(P(4))$. Moreover, (1) implies that $S(4) \hookrightarrow L_2(P(i))$ for $i = 1, 2, 3$ and $S(4) \hookrightarrow L_3(P(4))$.

Next, we want to claim that there exists some $i \geqslant 4$ such that $S(4) \hookrightarrow L_i(P(1))$, $S(4) \hookrightarrow L_i(P(2))$ and $S(4) \hookrightarrow L_i(P(3))$. By (1), $P(1)$ has a uniserial submodule U with $L_1(U) \cong S(0)$, $L_2(U) \cong S(4)$ and $L_3(U) = UJ^2 \cong S(1)$. On the other hand, $c(1, 0) = 1$ from (3.1)(i). Moreover, we have already got $S(0) \hookrightarrow L_3(P(1))$. Therefore, by [10, (1.1)Lemma], $S(4) \hookrightarrow L_i(P(1))$ for some $i \geqslant 4$. Thus, this holds for $P(2)$ and $P(3)$ as well by using the automorphism β in (3.1)(ii).

Therefore, we know so far the Loewy series of $P(1), \cdots, P(4)$ have at least the following form.

$$(2) \qquad P(j) \;=\; \begin{array}{c} S(j) \\ S(4)\cdots \\ S(0)\cdots \\ \vdots \\ S(4)\cdots \\ \vdots \\ S(j) \end{array} \qquad\qquad P(4) \;=\; \begin{array}{cccc} & & S(4) & \\ S(0) & S(1) & S(2) & S(3)\cdots \\ & & S(4)\cdots & \\ & & S(0)\cdots & \\ & & \vdots & \\ & & \vdots & \\ & & S(4) & \end{array}$$

for $j = 1, 2, 3$.

Assume that $\mathrm{Ext}^1_{kG}(S(1), S(2)) \neq 0$ and $\mathrm{Ext}^1_{kG}(S(1), S(3)) \neq 0$. Let $H = P(1) \cdot J/\mathrm{Soc}(P(1))$ be the heart of $P(1)$. Since $c(1, 2) = c(1, 3) = 1$ by (3.2)(i), the assumption and the self-duality of $S(0), \cdots, S(4)$ in (3.1)(ii) imply that $S(2)$ and $S(3)$ are both direct summands of H. Hence, it follows from (2) and the Cartan matrix for A in (3.1)(i) that the Loewy and socle series of $P(1)$ have the form

$$P(1) \quad = \quad \begin{array}{ccc} & S(1) & \\ & S(4) & \\ S(0) & S(2) & S(3). \\ & S(4) & \\ & S(1) & \end{array}$$

Thus, again (1.3) shows that $S(1) \hookrightarrow L_4(P(4))$, so that $S(i) \hookrightarrow L_4(P(4))$ for all $i = 1, 2, 3$ by using β. Hence $P(4)$ has Loewy series

$$P(4) \quad = \quad \begin{array}{c} S(4) \\ S(0) \quad S(1) \quad S(2) \quad S(3) \cdots \\ S(4) \quad \cdots \\ S(0) \quad S(1) \quad S(2) \quad S(3) \cdots \\ S(4) \end{array}$$

and there left only two $S(4)$'s form the Cartan matrix in (3.1)(i). Since $\mathrm{Ext}^1_{kG}(S(4), S(4)) = 0$ by (0), the only possibility for the Loewy series of $P(4)$ is that

$$P(4) \quad = \quad \begin{array}{c} S(4) \\ S(0) \quad S(1) \quad S(2) \quad S(3) \\ S(4) \quad S(4) \quad S(4) \\ S(0) \quad S(1) \quad S(2) \quad S(3) \\ S(4) \end{array} \quad .$$

Now, from the Loewy structure of $P(1)$ above, we know, by using the automorphism β again, that $P(4)$ has uniserial submodules U_1, U_2, U_3 of composition length 4 such that

$$U_1 = \begin{array}{c} S(1) \\ S(4) \\ S(0) \\ S(4) \end{array} \qquad U_2 = \begin{array}{c} S(2) \\ S(4) \\ S(0) \\ S(4) \end{array} \qquad U_3 = \begin{array}{c} S(3) \\ S(4) \\ S(0) \\ S(4). \end{array}$$

Hence, we can consider a submodule X of $P(4)$ defined by $X = U_1 + U_2 + U_3$. By (1), we have $\dim_k[\mathrm{Ext}^1_{kG}(S(0), S(4))] = 1$, which means that the multiplicity of $S(0)$ in $\mathrm{Soc}_2(X)/\mathrm{Soc}_1(X)$ is at most one. Hence, $\mathrm{Soc}_2(X)/\mathrm{Soc}_1(X) \cong S(0)$. Thus, since $\dim_k[\mathrm{Ext}^1_{kG}(S(4), S(0))] = 1$, we get that the multiplicity of $S(4)$ in $\mathrm{Soc}_3(X)/\mathrm{Soc}_2(X)$ is at most one Therefore, $\mathrm{Soc}_3(X)/\mathrm{Soc}_2(X) \cong S(4)$. Hence, X has Loewy and socle structure

$$X \quad = \quad \begin{array}{ccc} S(1) & S(2) & S(3) \\ & S(4) & \\ & S(0) & \\ & S(4) & \end{array} \quad .$$

So that, by (1.1) again, we know that the $S(1)$ in $L_1(X)$ comes from that in $L_2(P(4))$. Similar thing holds for $S(2)$ and $S(3)$ as well. Namely, it follows that $P(4)/X$ has Loewy series

$$P(4)/X \quad = \quad \begin{array}{ccccc} & & S(4) & & \\ & & S(0) & & \\ & S(4) & & S(4) & \\ S(1) & & S(2) & & S(3) \end{array} \quad .$$

This shows $\dim_k[\mathrm{Ext}^1_{kG}(S(0), S(4))] \geqslant 2$, contradicting (1).

Next, assume that $\mathrm{Ext}^1_{kG}(S(1), S(2)) \neq 0$ and $\mathrm{Ext}^1_{kG}(S(1), S(3)) = 0$. Then, by applying β^2 to $\mathrm{Ext}^1_{kG}(S(1), S(2))$, we get that $\mathrm{Ext}^1_{kG}(S(3), S(1)) \neq 0$, so that it follows $\mathrm{Ext}^1_{kG}(S(1), S(3)) \neq 0$ by the self-dualities, a contradiction. Similarly, we get a contradiction in the case that $\mathrm{Ext}^1_{kG}(S(1), S(2)) = 0$ and $\mathrm{Ext}^1_{kG}(S(1), S(3)) \neq 0$ by using β^2 in (3.2)(ii).

Therefore, it holds that $\mathrm{Ext}^1_{kG}(S(1), S(2)) = \mathrm{Ext}^1_{kG}(S(1), S(3)) = 0$. Then, (2) and the Cartan matrix in (3.1)(i) imply that $L_2(P(1)) \cong S(4)$, so that $P(1)$ has Loewy series of the form

$$(3) \qquad P(1) \quad = \quad \begin{array}{c} S(1) \\ S(4) \\ S(0) \cdots \\ \vdots \\ S(4) \cdots \\ \vdots \\ S(1) \end{array} \qquad \text{and there left } S(2),\ S(3).$$

Next, we want to claim $L_3(P(1)) \ncong S(0)$. Assume $L_3(P(1)) \cong S(0)$. Since $\mathrm{Ext}^1_{kG}(S(0), S(2)) = \mathrm{Ext}^1_{kG}(S(0), S(3)) = 0$ by (1), it follows from (3) that $L_4(P(1)) \cong S(4)$, which implies from (3) that $\mathrm{Ext}^1_{kG}(S(2), S(1)) \neq 0$, so that $\mathrm{Ext}^1_{kG}(S(1), S(2)) \neq 0$ by the self-dualities. This is a contradiction. Thus, $L_3(P(1)) \ncong S(0)$.

Suppose that $L_3(P(1)) \cong S(0) \bigoplus S(2)$. Since $\mathrm{Ext}^1_{kG}(S(3), S(1)) = 0$

by the self-dualities, we get by (3) that $P(1)$ has Loewy series of the form

$$P(1) \quad = \quad \begin{matrix} & S(1) & \\ & S(4) & \\ S(0) & & S(2) \\ & S(3) & \\ & S(4) & \\ & S(1) & \end{matrix} \quad .$$

Let $V = [P(1)\cdot J^3]^*$. Then, by the self-dualities, V is a uniserial kG-module of composition length three with $L_1(V) \cong S(1)$, $L_2(V) \cong S(4)$, $L_3(V) = VJ^2 \cong S(3)$, which means that $S(3) \hookrightarrow L_3(P(1))$, contradicting the Loewy structure of $P(1)$ above. Hence, $L_3(P(1)) \ncong S(0) \bigoplus S(2)$.

Similarly, we obtain that $L_3(P(1)) \ncong S(0) \bigoplus S(3)$. Therefore, it follows that $L_3(P(1)) \cong S(0) \bigoplus S(2) \bigoplus S(3)$ by (3), so that we completely know the Loewy structure of $P(1)$. Thus, we get the Loewy and socle structure of $P(1)$, $P(2)$ and $P(3)$ as in the statement by making use of β. Hence, again by (1.3) and the Cartan matrix in (3.1)(i), $P(4)$ has Loewy series of the form

$$P(4) \quad = \quad \begin{matrix} & & S(4) & & \\ S(0) & S(1) & & S(2) & S(3)\cdots \\ & & S(4)\cdots & & \\ S(0) & S(1) & & S(2) & S(3)\cdots \\ & & S(4) & & \end{matrix} \quad .$$

and there left only two $S(4)$'s. Since $\mathrm{Ext}^1_{kG}(S(4), S(4)) = 0$ by (0), we finally get the complete Loewy series of $P(4)$ as in the statement. This finishes the proof of the theorem. Q.E.D.

Acknowledgements.

The first author was in part supported by the Joint Research Project "Representation Theory of Finite and Algebraic Groups" 1997–99 under the Japanese-German Cooperative Science Promotion Program supported by JSPS and DFG.

References

[1] M. Auslander, I. Reiten and S.O. Smalø, Representation Theory of Artin Algebras, Cambridge Univ. Press, Cambridge.

[2] J. Brandt, A lower bound for the number of irreducible characters in a block, J. Algebra, **74** (1982), 509–515.

[3] L. Dornhoff, Group Representation Theory (part B), Marcel Dekker, New York.

[4] K. Erdmann, On Auslander-Reiten components for group algebras, J. Pure and Appl. Algebra, **109** (1995), 149–160.

[5] M. Geck, Irreducible Brauer characters of the 3-dimensional special unitary groups in non-defining characteristic, Commun. Algebra, **18** (1990), 563–584.

[6] K. Hicks, The Loewy structure and basic algebra structure for some linebreak-three dimensional projective special unitary groups in characteristic 3, J. Algebra, **202** (1998), 192–201.

[7] B. Huppert, Endliche Gruppen I, Springer-Verlag, Berlin.

[8] S. Kawata, On Auslander-Reiten components for certain group modules, Osaka J. Math., **30** (1993), 137–157.

[9] M. Klemm, Charakterisierung der Gruppen $PSL(2, p^f)$ and $PSU(3, p^{2f})$ durch ihre Charaktertafel, J. Algebra, **24** (1973), 127–153.

[10] S. Koshitani, On the Loewy series of the group algebra of a finite p-solvable group with p-length > 1, Commun. Algebra, **13** (1985), 2175–2198.

[11] P. Landrock, The Cartan matrix of a group algebra modulo any power of its radical, Proc. Amer. Math. Soc., **88** (1983), 205–206.

[12] P. Landrock, Finite Group Algebras and Their Modules, London Math. Soc. Lecture Note Series, Cambridge Univ. Press, Cambridge.

[13] H. Nagao and Y. Tsushima, Representations of Finite Groups, Academic Press, New York.

[14] W.A. Simpson and J.S. Frame, The character tables for $SL(3, q)$, $SU(3, q^2)$, $PSL(3, q)$, $PSU(3, q^2)$, Canad. J. Math., **25** (1973), 486–494.

[15] P. Webb, The Auslander-Reiten quiver of a finite group, Math. Z., **179** (1982), 97–121.

Shigeo Koshitani
Department of Mathematics and Informatics
Faculty of Science, Chiba University
Chiba 263-8522, Japan
e-mail: koshitan@math.s.chiba-u.ac.jp

Naoko Kunugi
Department of Mathematics, Graduate School of Science and Technology
Chiba University, Chiba 263-8522, Japan
e-mail: mkunugi@g.math.s.chiba-u.ac.jp