Advanced Studies in Pure Mathematics 23, 1994
Spectral and Scattering Theory and Applications
pp. 295–305

Asymptotic Behavior of Solutions for the Coupled Klein-Gordon-Schrödinger Equations

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Dedicated to Professor S.T. Kuroda on his 60th birthday

§1. Introduction and theorem

In the present paper we consider the asymptotic behavior in time of solutions for the coupled Klein-Gordon-Schrödinger equations:

$$(1.1)\qquad i\frac{\partial}{\partial t}\psi + \frac{1}{2}\Delta\psi = \phi\psi, \quad t \in \mathbf{R}, \quad x \in \mathbf{R}^N,$$

$$(1.2)\qquad \frac{\partial^2}{\partial t^2}\phi - \Delta\phi + \phi = -|\psi|^2, \quad t \in \mathbf{R}, \quad x \in \mathbf{R}^N,$$

$$(1.3)\qquad \psi(0,x) = \psi_0(x), \quad \phi(0,x) = \phi_0(x), \quad \frac{\partial}{\partial t}\phi(0,x) = \phi_1(x).$$

Equations (1.1)–(1.2) describe a classical model of Yukawa's interaction of conserved complex nucleon field with neutral real meson field and the associated mass has been normalized as unity. Here ψ is a complex scalar nucleon field, and ϕ is a real scalar meson field. (1.1)–(1.2) are a semi-relativistic version of the coupled Klein-Gordon-Dirac equations (see, e.g., [2]).

Since the interaction above is only quadratic, the problems concerning asymptotic behavior of solutions are harder than the cases of higher interactions, especially in lower space dimensions. In order to examine the basic structure of nonlinearities of (1.1)–(1.2), it would be instructive to look at the decoupled case with self-interaction.

There are a large amount of papers concerning the asymptotic behavior in time of solutions for the nonlinear Schrödinger equation

$$(1.4)\qquad i\frac{\partial}{\partial t}u + \frac{1}{2}\Delta u = |u|^{p-1}u, \quad ,t \in \mathbf{R}, \quad x \in \mathbf{R}^N,$$

Received December 8, 1992.

and the nonlinear Klein-Gordon equation

$$\frac{\partial^2}{\partial t^2}u - \Delta u + u = -|u|^{p-1}u, \quad t \in \mathbf{R}, \quad x \in \mathbf{R}^N \tag{1.5}$$

(for the nonlinear Schrödinger equation, see [3], [6], [11]–[13], [16], [17], [19], [21], [23], [27]–[31] and for the nonlinear Klein-Gordon equation, see [4], [5], [14], [18], [20], [22], [24], [26]–[29]). When we consider the asymptotic behavior of solutions for (1.4) or (1.5), it is natural and important to investigate whether the wave operators $W_\pm$ exist or not. For (1.4), we define the wave operator W_+ as follows. Let $u_+(t)$ be the solution of the free Schrödinger equation

$$i\frac{\partial}{\partial t}u + \frac{1}{2}\Delta u = 0, \quad , t \in \mathbf{R}, \quad x \in \mathbf{R}^N, \tag{1.6}$$

with $u_+(0) = \psi_+$. If one can look for the solution $u(t)$ of (1.4) with $u(0) = \psi_0$ such that $u(t)$ exists globally in time and

$$\|u_+(t) - u(t)\|_{L^2} \longrightarrow 0 \quad (t \to +\infty), \tag{1.7}$$

then the wave operator W_+ can be defined by a mapping from $u_+(0) = \psi_+$ to $u(0) = \psi_0$. Here ψ_+ and ψ_0 are called a scattered state and an interacting state, respectively. For the case of $t \to -\infty$, the wave operator W_- is defined in the same way. We can also consider the wave operators $W_\pm$ for (1.5). In [6], [26] and [31] it is proved that when $N = 3$ and the nonlinear term is quadratic, that is, $p = 2$, both in the cases (1.4) and (1.5) the wave operators $W_\pm$ can be defined for some data. On the other hand, in [3], [10], [14], [17] and [20] it is proved that when $N = 1, 2$ and $p = 2$, there exist no nontrivial asymptotically free solutions for (1.4) and (1.5), that is, the wave operators $W_\pm$ can not be defined for any nonzero data. This is because the time decay rate of solutions of (1.4) and (1.5) for $N = 2$ is worse than that for $N = 3$. Therefore, we have to consider the modified wave operators for (1.4) and (1.5) with $N = 2$ and $p = 2$ (see, e.g., [23]).

The unique global existence of solutions for (1.1)–(1.3) are already established (see [1], [2], [8] and [15]). Fukuda and M. Tsutsumi [9] and Strauss [29] studied the asymptotic behavior as $t \to \pm\infty$ of solutions for the coupled Klein-Gordon-Schrödinger equations with interactions higher than the quadratic order of (1.1)–(1.2). The results in [9] and [29] are similar to the results obtained for the decoupled nonlinear Klein-Gordon and Schrödinger equations.

If there is a complete analogy between the full system (1.1)–(1.2) and the decoupled system (1.4) and (1.5), it is natural to conjecture that when $N = 2$, the wave operators $W_\pm$ could not be defined for (1.1)–(1.2). But this conjecture is not true. The purpose in the present paper is to show that when $N = 2$, the wave operators $W_\pm$ for (1.1)–(1.2) can be defined for certain scattered data.

This is a sharp contrast to the decoupled case and gives a reason that the coupled Klein-Gordon-Schrödinger equations are not a simple superposition of the nonlinear Klein-Gordon and Schrödinger equations.

Before we state the theorem, we define several notations. Let $\omega = \sqrt{-\Delta + 1}$ and let $U(t) = e^{\frac{i}{2}t\Delta}$ be the evolution operator of the free Schrödinger equation. We denote by $\hat{f}$ the Fourier transform of f. For nonnegative integers m and s, we define H^m and $H^{m,s}$ as follows:

$$
\begin{aligned}
H^m &= \{v \in \mathcal{S}'(\mathbf{R}^N); \|(1-\Delta)^{\frac{m}{2}} v\|_{L^2} < +\infty\}, \\
H^{m,s} &= \{v \in \mathcal{S}'(\mathbf{R}^N); \|(1+|x|^2)^{\frac{s}{2}}(1-\Delta)^{\frac{m}{2}} v\|_{L^2} < +\infty\}
\end{aligned}
$$

with the norms

$$
\begin{aligned}
\|v\|_{H^m} &= \|(1-\Delta)^{\frac{m}{2}} v\|_{L^2}, \\
\|v\|_{H^{m,s}} &= \|(1+|x|^2)^{\frac{s}{2}}(1-\Delta)^{\frac{m}{2}} v\|_{L^2},
\end{aligned}
$$

respectively. For a multi-index $\alpha = (\alpha_1, \cdots \alpha_N)$ with nonnegative integers α_j, we put

$$
\begin{aligned}
|\alpha| &= \alpha_1 + \cdots + \alpha_N, \\
(\frac{\partial}{\partial x})^\alpha &= (\frac{\partial}{\partial x_1})^{\alpha_1} \cdots (\frac{\partial}{\partial x_N})^{\alpha_N}.
\end{aligned}
$$

For $p \geq 1$ and a nonnegative integer k, we let

$$
W^{k,p} = \{u \in L^p(\mathbf{R}^N); (\frac{\partial}{\partial x})^\alpha u \in L^p(\mathbf{R}^N), \quad |\alpha| \leq k\}
$$

with the norm

$$
\|u\|_{W^{k,p}} = \sum_{|\alpha| \leq k} \|(\frac{\partial}{\partial x})^\alpha u\|_{L^p}.
$$

We now state the theorem.

Theorem 1.1. *Let $N = 2$ and $\varepsilon > 0$.*

(i) *Assume that $\psi_+ \in H^{2,4}$, $(1+|x|^2)^j (\frac{\partial}{\partial x})^\alpha \psi_+ \in L^1(\mathbf{R}^2)$ for $j + |\alpha| \leq 2$ and $\operatorname{supp} \hat{\psi}_+ \subset \{\xi; |\xi| \geq 1 + \varepsilon\}$. We put $u_+(t) = e^{\frac{i}{2}t\Delta}\psi_+$. Assume that $\phi_{+0} \in H^{4,2}$, $\phi_{+1} \in H^{3,2}$, for $|\alpha| \leq 4$ $(\frac{\partial}{\partial x})^\alpha \phi_{+0} \in L^1(\mathbf{R}^2)$,* *and for* $|\alpha| \le 3$ $(\frac{\partial}{\partial x})^\alpha \phi_{+1} \in L^1(\mathbf{R}^2)$. *We put* $v_+(t) = (\cos \omega t)\phi_{+0} + (\omega^{-1} \sin \omega t)\phi_{+1}$. *Then, there exists* $\eta > 0$ *such that if*

$$(1.8) \quad \|\psi_+\|_{H^{2,4}} + \sum_{j+|\alpha|\le 2} \|(1+|x|)^j (\frac{\partial}{\partial x})^\alpha \psi_+\|_{L^1} + \|\phi_{+0}\|_{H^{4,2}} + \|\phi_{+0}\|_{W^{4,1}} + \|\phi_{+1}\|_{H^{3,2}} + \|\phi_{+1}\|_{W^{3,1}} \le \eta,$$

(1.1)–(1.2) *have the unique solutions* (ψ, ϕ) *satisfying*

$$(1.9) \quad \psi \in \bigcap_{j=0}^{1} C^j([0,\infty); H^{2-2j}),$$

$$(1.10) \quad \phi \in \bigcap_{j=0}^{2} C^j([0,\infty); H^{2-j}),$$

$$(1.11) \quad \|\psi(t) - u_+(t)\|_{H^2} + \|\phi(t) - v_+(t)\|_{H^2} + \|\frac{\partial}{\partial t}\phi(t) - \frac{\partial}{\partial t} v_+(t)\|_{H^1} = O(t^{-1}) \quad (t \to +\infty),$$

$$(1.12) \quad (\int_t^{+\infty} \|\psi(s) - u_+(s)\|_{W^{2,4}}^4 ds)^{1/4} = O(t^{-1}) \quad (t \to +\infty),$$

where η *depends only on* ε.

(ii) *Assume that* $\psi_- \in H^{2,4}$, $(1+|x|^2)^j (\frac{\partial}{\partial x})^\alpha \psi_- \in L^1(\mathbf{R}^2)$ *for* $j + |\alpha| \le 2$ *and* $\operatorname{supp} \hat{\psi}_- \subset \{\xi; |\xi| \ge 1 + \varepsilon\}$. *We put* $u_-(t) = e^{\frac{i}{2}t\Delta}\psi_-$. *Assume that* $\phi_{-0} \in H^{4,2}$, $\phi_{-1} \in H^{3,2}$, *for* $|\alpha| \le 4$ $(\frac{\partial}{\partial x})^\alpha \phi_{-0} \in L^1(\mathbf{R}^2)$, *and for* $|\alpha| \le 3$ $(\frac{\partial}{\partial x})^\alpha \phi_{-1} \in L^1(\mathbf{R}^2)$. *We put* $v_-(t) = (\cos \omega t)\phi_{-0} + (\omega^{-1} \sin \omega t)\phi_{-1}$. *Then, there exists* $\eta > 0$ *such that if*

$$(1.13) \quad \|\psi_-\|_{H^{2,4}} + \sum_{j+|\alpha|\le 2} \|(1+|x|)^j (\frac{\partial}{\partial x})^\alpha \psi_-\|_{L^1} + \|\phi_{-0}\|_{H^{4,2}} + \|\phi_{-0}\|_{W^{4,1}} + \|\phi_{-1}\|_{H^{3,2}} + \|\phi_{-1}\|_{W^{3,1}} \le \eta,$$

(1.1)–(1.2) *have the unique solutions* (ψ, ϕ) *satisfying*

$$(1.14) \quad \psi \in \bigcap_{j=0}^{1} C^j([0,\infty); H^{2-2j}),$$

$$(1.15) \quad \phi \in \bigcap_{j=0}^{2} C^j([0,\infty); H^{2-j}),$$

$$(1.16) \quad \|\psi(t) - u_-(t)\|_{H^2} + \|\phi(t) - v_-(t)\|_{H^2} + \|\frac{\partial}{\partial t}\phi(t) - \frac{\partial}{\partial t}v_-(t)\|_{H^1} = O(t^{-1}) \quad (t \to -\infty),$$

$$(1.17) \quad (\int_t^{+\infty} \|\psi(s) - u_-(s)\|_{W^{2,4}}^4 ds)^{1/4} = O(t^{-1}) \quad (t \to -\infty),$$

where η depends only on ε.

Remark. The unique global existence theorem for the Cauchy problem of (1.1)–(1.3) is already established (see [1], [2], [8] and [15]). In [1], [2], [8] and [15] only the case of $N = 3$ is treated, but the proof of the unique global solutions for $N = 2$ is easier than that for $N = 3$. Therefore, the solutions of (1)–(2) given by (i) and (ii) of Theorem 1.1 can be extended to the whole time interval $(-\infty, +\infty)$.

The following corollary is an immediate consequence of Theorem 1.1.

Corollary 1.2. *Assume $N = 2$. Let $\varepsilon > 0$.*

(i) *By D_+ we denote the set of all scattered states $(\psi_+, \phi_{+0}, \phi_{+1})$ such that* $\operatorname{supp} \hat{\psi}_+ \subset \{\xi; |\xi| \geq 1 + \varepsilon\}$ *and* (1.8) *holds. Then, for* (1.1)–(1.2) *the wave operator $W_+ : (\psi_+, \phi_{+0}, \phi_{+1}) \mapsto (\psi(0), \phi(0), \frac{\partial}{\partial t}\phi(0))$ is well defined on D_+.*

(ii) *By D_- we denote the set of all scattered states $(\psi_-, \phi_{-0}, \phi_{-1})$ such that* $\operatorname{supp} \hat{\psi}_- \subset \{\xi; |\xi| \geq 1 + \varepsilon\}$ *and* (1.13) *holds. Then, for* (1.1)–(1.2) *the wave operator $W_- : (\psi_-, \phi_{-0}, \phi_{-1}) \mapsto (\psi(0), \phi(0), \frac{\partial}{\partial t}\phi(0))$ is well defined on D_-.*

The proofs in the previous papers [9] and [29] are the same as those used for (1.4) and (1.5) and do not have anything to do with the specific feature of quadratic nonlinearities. Our proof of Theorem 1.1 is based on the special property of the Yukawa interaction and on the improved decay estimates of the interaction term which take account of the difference between the propagation properties of the Schrödinger wave and the Klein-Gordon wave.

§2. Sketch of Proof of Theorem 1.1

We first summarize several lemmas needed for the proof of Theorem 1.1 without proofs.

Lemma 2.1. *Let $N \geq 1$.*

(i) *Let p and q be two positive constants such that* $1/p+1/q=1$ *and* $2 \le p \le +\infty$. *Then,*

$$\|U(t)v\|_p \le (2\pi|t|)^{-N/2+N/p}\|v\|_q, \quad v \in L^q, \quad t \ne 0. \tag{2.1}$$

(ii) *Let k be a nonnegative integer. Suppose that for* $j+|\alpha| \le k$ $(1+|x|)^{j+2}\left(\frac{\partial}{\partial x}\right)^\alpha \psi \in L^2$ *and* $(1+|x|)^{j+2}\left(\frac{\partial}{\partial x}\right)^\alpha \psi \in L^1$. *We put* $u_0(t,x) = e^{i|x|^2/(2t)}(it)^{-N/2}\hat{\psi}(\frac{x}{t})$. *Then, for some* $K>0$,

$$\sum_{2j+|\alpha|\le k} \|\left(\frac{\partial}{\partial x}\right)^\alpha \left(\frac{\partial}{\partial t}\right)\{U(t)\psi - u_0(t)\}\|_2 \le K|t|^{-1} \sum_{j+|\alpha|\le k} \|(1+|x|)^{j+2}\left(\frac{\partial}{\partial x}\right)^\alpha \psi\|_2, \quad |t| \ge 1, \tag{2.2}$$

$$\sum_{2j+|\alpha|\le k} \|\left(\frac{\partial}{\partial x}\right)^\alpha \left(\frac{\partial}{\partial t}\right)\{U(t)\psi - u_0(t)\}\|_\infty \le K|t|^{-N/2-1} \sum_{j+|\alpha|\le k} \|(1+|x|)^{j+2}\left(\frac{\partial}{\partial x}\right)^\alpha \psi\|_2, \quad |t| \ge 1, \tag{2.3}$$

where K depends only on k and N.

For the proof of Lemma 2.1, see, e.g., [33, Lemmma 2.1].

Lemma 2.2. *Assume* $N=2$. *Let k be a nonnegative integer. Then, for some* $K>0$,

$$\sum_{j+|\alpha|\le k} \|\left(\frac{\partial}{\partial x}\right)^\alpha \left(\frac{\partial}{\partial t}\right)^j (\cos \omega t)v\|_\infty \le K(1+|t|)^{-1}(\|v\|_{W^{2+k,1}} + \|v\|_{H^{2+k}}), \quad t \in \mathbf{R}, \tag{2.4}$$

$$\sum_{j+|\alpha|\le k} \|\left(\frac{\partial}{\partial x}\right)^\alpha \left(\frac{\partial}{\partial t}\right)^j (\omega^{-1}\sin \omega t)v\|_\infty \le K(1+|t|)^{-1}(\|v\|_{W^{1+k,1}} + \|v\|_{H^{1+k}}), \quad t \in \mathbf{R}, \tag{2.5}$$

where K depends only on k.

For the proof of Lemma 2.2, see, e.g., [5], [13] and [22].

We next state the decay estimate of solution for the Klein-Gordon equation outside of the light cone.

Lemma 2.3. *Assume $N \geq 1$. Let $\varepsilon > 0$ and let k be a nonnegative integer. Then, for some $L > 0$,*

$$\sum_{j+|\alpha|\leq k} \|(\frac{\partial}{\partial x})^{\alpha}(\frac{\partial}{\partial t})^{j}(\cos\omega t)v\|_{L^{\infty}(|x|>(1+\varepsilon)|t|)} \leq L(1+|t|)^{-2}\|v\|_{H^{k+[N/2]+1,2}}, \quad t \in \mathbf{R}, \tag{2.6}$$

$$\sum_{j+|\alpha|\leq k} \|(\frac{\partial}{\partial x})^{\alpha}(\frac{\partial}{\partial t})^{j}(\omega^{-1}\sin\omega t)v\|_{L^{\infty}(|x|>(1+\varepsilon)|t|)} \leq L(1+|t|)^{-2}\|v\|_{H^{k+[N/2],2}}, \quad t \in \mathbf{R}, \tag{2.7}$$

where $[N/2]$ is the largest integer that does not exceed $N/2$, and L depends only on ε, k and N.

The proof of Lemma 2.3 is based on the finite speed propagation of the Klein-Gordon wave. For the details, see, e.g., [25, Theorem XI. 17] and [33, Lemma 2.3].

We next consider the following problem: Given $h(t)$, find $u(t)$ such that

$$\frac{\partial^2}{\partial t^2}u - \Delta u + u = h(t), \quad t \geq 0, \quad x \in \mathbf{R}^N, \tag{2.8}$$

$$\|\frac{\partial}{\partial t}u(t)\|_2^2 + \|\nabla u(t)\|_2^2 + \|u(t)\|_2^2 \longrightarrow 0 \quad (t \to +\infty), \tag{2.9}$$

or

$$\frac{\partial^2}{\partial t^2}u - \Delta u + u = h(t), \quad t \leq 0, \quad x \in \mathbf{R}^N, \tag{2.10}$$

$$\|\frac{\partial}{\partial t}u(t)\|_2^2 + \|\nabla u(t)\|_2^2 + \|u(t)\|_2^2 \longrightarrow 0 \quad (t \to -\infty). \tag{2.11}$$

We assume that for some $M > 0$,

$$\sup_{t\in[0,\infty)} \left[(1+t)\|h(t)\|_2 + (1+t)^2 \sum_{1\leq j+|\alpha|\leq 3} \|(\frac{\partial}{\partial x})^{\alpha}(\frac{\partial}{\partial t})h(t)\|_2\right] \leq M, \tag{2.12}$$

$$\sup_{t\in(-\infty,0]} \left[(1-t)\|h(t)\|_2 + (1-t)^2 \sum_{1\leq j+|\alpha|\leq 3} \|(\frac{\partial}{\partial x})^{\alpha}(\frac{\partial}{\partial t})h(t)\|_2\right] \leq M. \tag{2.13}$$

We have the following lemma concerning the existence of solution for (2.8)–(2.9) and (2.10)–(2.11).

Lemma 2.4. *Let $N \geq 1$.*

(i) *Assume that $h \in \cap_{j=0}^{3} C^j([0,\infty); H^{3-j})$ and that $h(t)$ satisfies* (2.12)*. Then, there exists a unique solution $u(t)$ of* (2.8)–(2.9) *such that*

$$u \in \bigcap_{j=0}^{4} C^j([0,\infty); H^{4-j}), \tag{2.14}$$

$$\sup_{t\in[0,\infty)} (1+t) \sum_{j+|\alpha|\leq 4} \|\Big(\frac{\partial}{\partial x}\Big)^{\alpha}\Big(\frac{\partial}{\partial t}\Big)^{j} u(t)\|_2 \leq C_0 M, \tag{2.15}$$

where M is defined (2.12) *and C_0 is independent of h and u.*

(ii) *Assume that $h \in \cap_{j=0}^{3} C^j((-\infty,0]; H^{3-j})$ and that $h(t)$ satisfies* (2.13)*. Then, there exists a unique solution $u(t)$ of* (2.10)–(2.11) *such that*

$$u \in \bigcap_{j=0}^{4} C^j((-\infty,0]; H^{4-j}), \tag{2.16}$$

$$\sup_{t\in(-\infty,0]} (1-t) \sum_{j+|\alpha|\leq 4} \|\Big(\frac{\partial}{\partial x}\Big)^{\alpha}\Big(\frac{\partial}{\partial t}\Big)^{j} u(t)\|_2 \leq C_0 M, \tag{2.17}$$

where M is defined (2.13) *and C_0 is independent of h and u.*

For the proof of Lemma 2.4, see [33, Lemma 2.4].

Now we describe a sketch of proof of Theorem 1.1. We consider only the case of $t \to +\infty$, because the proof for the case of $t \to -\infty$ is quite similar to that for the case of $t \to +\infty$.

We seek the solutions to the final value problem for (1.1)–(1.2) around almost free solutions. We choose a function $z \in C^{\infty}([0,\infty))$ such that $z(t) = 1$ for $t \geq 2$ and $z(t) = 0$ for $0 \leq t \leq 1$. We put

$$u_1(t) = z(t)u_0(t) = z(t)e^{i|x|^2/(2t)}(it)^{-1}\hat{\psi}_+\Big(\frac{x}{t}\Big), \tag{2.18}$$

where $u_0(t,x)$ is defined in Lemma 2.1 (ii). Let $v_0(t,x)$ be a solution of (2.8)–(2.9) given by Lemma 2.4 (i) with $h = |u_1|^2$. We introduce the following almost free solutions.

$$u(t) = U(t)\psi_+, \quad v(t) = (\cos\omega t)\phi_{+0} + (\omega^{-1}\sin\omega t)\phi_{+1} + v_0(t). \tag{2.19}$$

We note that $u(t) = u_+(t)$. Furthermore, we put

$$\psi(t) = F(t) + u(t), \tag{2.20}$$

$$\phi(t) = N(t) + v(t). \tag{2.21}$$

We rewrite (1.1)–(1.2) as the following system of F and N:

$$i\frac{\partial}{\partial t}F + \frac{1}{2}\Delta F = NF + N(u - u_1) + Nu_1 \\ + vF + f(t), \quad t \geq 0, \quad x \in \mathbf{R}^2, \tag{2.22}$$

$$\frac{\partial^2}{\partial t^2}N - \Delta N + N = |F|^2 + 2\Re(F(\bar{u} - \bar{u}_1)) \\ + 2\Re(F\bar{u}_1) + g(t), \quad t \geq 0, \quad x \in \mathbf{R}^2, \tag{2.23}$$

$$\|F(t)\|_2 \to 0 \quad (t \to \infty), \tag{2.24}$$

$$\|\frac{\partial}{\partial t}N(t)\|_2^2 + \|\nabla N(t)\|_2^2 + \|N(t)\|_2^2 \to 0 \quad (t \to \infty), \tag{2.25}$$

where

$$f(t) = v(u - u_1) + vu_1, \tag{2.26}$$

$$g(t) = |u - u_1|^2 + 2\Re((u - u_1)\bar{u}_1). \tag{2.27}$$

If we have the solutions (F, N) of (2.22)–(2.25), then we obtain Theorem 1.1 (i) by (2.20)–(2.21). Lemmas 2.1–2.4 and the support condition of the Fourier image of ψ_+ show that $f(t)$ and $g(t)$ decay in L^2 fast enough as $t \to \infty$. Therefore, we can obtain the desired solutions (F, N) for (2.22)–(2.25). The details of the proof will be published elsewhere (see [33]).

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