Raghu Raj Bahadur 1924-1997

By Stephen M. Stigler

Raghu Raj Bahadur was born in Delhi, India on April 30, 1924. He was extremely modest in demeanor and uncomfortable when being honored, but on several occasions the Chicago Department of Statistics managed to attract him to birthday celebrations by taking advantage of the coincidence of his and Gauss's birthdates -- he would come to honor Gauss, not to receive honor for himself. At St. Stephen's College of Delhi University he excelled, graduating in 1943 with first class honors in mathematics. In 1944 he won a scholarship and generously returned the money to the College to aid poor students. But it is clear that he had not yet found his calling. In that year, 1944, his was judged the best serious essay by a student in the College. The essay gave no hint of the career that was to follow: it was a somber essay, on the isolation of individuals, and it gave a dark and pessimistic view of the search for meaning in life – a vision that was foreign to the Raj we knew in later years. He continued on at Delhi, receiving a Masters degree in mathematics in 1945. After a year at the Indian Institute of Science in Bangalore he was awarded a scholarship by the government of India for graduate studies, and in October 1947, after spending one year at the Indian Statistical Institute in Calcutta, Raj took an unusual and fateful step. While India was in the upheaval that followed partition and preceded independence, Raj traveled to Chapel Hill, North Carolina, to study mathematical statistics.

In barely over two years he completed his Ph.D. His dissertation focused on decision theoretic problems for *k* populations, a problem suggested by Harold Hotelling (although Herbert Robbins served as his major professor). In a December 1949 letter of reference, Harold Hotelling wrote: "His thesis, which is now practically complete, includes for one thing a discussion of the following paradox: Two samples are known to be from Cauchy populations

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whose central values are known, but it is not known which is which. Probability of erroneous assignment of the samples to the two populations may be larger in some cases when the greater sample mean is ascribed to the greater population mean than when the opposite is done." His first paper, including this example, was published in the *Annals of Mathematical Statistics* (1).

At the winter statistical meetings in December 1949, W. Allen Wallis contacted him to sound him out – was he interested in joining the new group of statisticians being formed at the University of Chicago? He was interested, and Allen arranged for Raj to start in the Spring Quarter of 1950. Raj's move to Chicago was to prove a pivotal event in his life. He left Chicago twice (in 1952 and in 1956), and he returned twice (in 1954 and in 1961). He never forgot his roots in India, and the pull of family and the intellectual community in Delhi caused him to return there time and again throughout his life, but Chicago had a special, irresistible allure for him. In the decade following 1948, Allen Wallis assembled an extraordinarily exciting and influential intellectual community. Starting with Jimmie Savage, William Kruskal, Leo Goodman, Charles Stein, and Raj Bahadur, he soon added David Wallace, Paul Meier, and Patrick Billingsley.

Raj thrived at Chicago, although sometimes the pric.e was high. One of his great achievements was his 1954 paper on "Sufficiency and Statistical Decision Functions," (5) a monumental paper (it ran to 40 pages in the *Annals*) that is a masterpiece of both mathematics and statistics. The story of its publication tells much about the atmosphere in Chicago in those days. It was originally submitted in May of 1952, and, with Raj away in India, it was assigned to Jimmie Savage as a referee. Savage was favorable and impressed, and fairly quick in his report (he took two months on what must have been a 100 page manuscript of dense mathematics), so why was there a two year delay in publication? It was not because of a backlog; the *Annals* was

publishing with a three month delay in those days. Rather it was the character of the report and the care of Raj's response. For while Savage was favorable, his reports (eventually there were three) ran to 20 single-spaced pages, asking probing questions as well as listing over 60 points of linguistic and mathematical style. Somehow Raj survived this barrage, rewriting the paper completely, benefiting from the comments but keeping the work his own, and preserving, over another referee's objections, an expository style that explained the deep results both as mathematics and again as statistics.

From 1956-61 Raj was again in India, this time as a Research Statistician at the Indian Statistical Institute, Calcutta, but in 1961 he returned to the University of Chicago to stay, except for two leaves back to India. He retired in 1991 but continued to take vigorous part in the intellectual life of the Department as long as his increasingly frail health permitted. He died on June 7, 1997.

Bahadur's research in the 1950s and 1960s played a fundamental role in the development of mathematical statistics over that period. These works included a series of papers on sufficiency (5)-(8), investigations on the conditions under which maximum likelihood estimators will be consistent (including Bahadur's Example of Inconsistency) (12), new methods for the comparison of statistical tests (including the measure based upon the theory of large deviations now known as Bahadur Efficiency) (16,17), and an approach to the asymptotic theory of quantiles (now recognized as the Bahadur Representation of Sample Quantiles) (25). C. R. Rao has written "Bahadur's theorem [his 1957 converse to the Rao-Blackwell theorem (11)] is one of the most beautiful theorems of mathematical statistics" [in *Glimpses of India's Statistical Heritage*, Ed. J. K. Ghosh, S. K. Mitra, K. R. Parthasarathy, Wiley Eastern, 1992, p. 162]. Other work included his approach to classification of responses from dichotomous

questionnaires (including the Bahadur-Lazarsfeld Expansion) (20, 21), and the asymptotic optimality of the likelihood ratio test in a large deviation sense (24). Bahadur summarized his research in the theory of large deviations in an elegant short monograph, *Some Limit Theorems in Statistics* (30).

Virtually everything Raj did was characterized by a singular depth and elegance. He took particular pleasure in showing how simplicity and greater generality could be allies rather than antagonists, as in his demonstration that LeCam's theorem on Fisher's bound for asymptotic variances could be derived from a clever appeal to the Neyman-Pearson Lemma (23). Raj's work was remarkable for its elegance and deceptive simplicity. He forever sought the "right" way of approaching a subject – a combination of concept and technique that not only yielded the result but also showed precisely how far analysis could go. Isaac Newton labored hard to draw the right diagram, to outline in simple steps a demonstration that made the most deep and subtle principles of celestial mechanics seem clear and unavoidably natural. Raj had a similar touch in mathematical statistics. His own referee's reports were minor works of art; his papers often masterpieces.

In the early 1950s he married Thelma Clark, and together they raised two fine children, Sekhar and Sheila Ann, of whom they were immensely proud. From his first arrival in 1950 for the rest of his life, Raj felt Chicago was a precious place. He evidently found here, in the intellectual life of the Department and the close companionship of his family, the meaning he had been seeking when he wrote his somber essay in 1944.

Raj Bahadur was President of the IMS in 1974-75, and he was the IMS's 1974 Wald Lecturer. He was honored by the Indian Society for Probability and Statistics in November

1987. In 1993 a Festschrift was published in his honor, *Statistics and Probability*, edited by J. K. Ghosh, S. K. Mitra, K. R. Parthasarathy, and B. L. S. Prakasa Rao (Wiley Eastern).

Raghu Raj Bahadur

Born April 30, 1924, Delhi, India; died June 7, 1977, Chicago, Illinois.

Education

B.A. (Honours)	Mathematics (with Physics)	Delhi University (St. Stephen's College)	1943
M.A.	Mathematics	Delhi University (St. Stephen's College)	1945
Ph.D.	Mathematical Statistics	North Carolina University (Chapel Hill)	1950

Professional Career

Research Associate in Applied Statistics, Indian Statistical Institute, Calcutta, 1946-47.
Research Associate in Mathematical Statistics, University of North Carolina, 1949-50.
Instructor in Statistics, University of Chicago, 1950-51.
Professor of Statistics, Indian Council of Agricultural Research, New Delhi, 1952-53.
Visiting Assistant Professor of Mathematical Statistics, Columbia University, 1953-54.
Assistant Professor of Statistics, University of Chicago, 1954-56.
Research Statistician, Indian Statistical Institute, Calcutta, 1956-61.
Associate Professor of Statistics, University of Chicago, 1961-65.
Professor of Statistics, University of Chicago, 1965-91.
Distinguished Visiting Professor, Indian Statistical Institute, 1972-97.
Professor Emeritus, University of Chicago, 1992-97.

Professional Memberships

Fellow, Institute of Mathematical Statistics.
Member, International Statistical Institute.
Fellow, Indian National Sciences Academy.
Fellow, Indian Academy of Sciences.

Professional Activities and Honors

John Simon Guggenheim Fellow, 1968-69.
Ten lectures on limit theorems in statistics at the SIAM regional conference at Tallahassee, Florida, in 1969.
Associate Editor, *Annals of Mathematical Statistics,* 1964-1973.
Member, Council of the Indian Statistical Institute, 1972-74.
Member, Editorial Board of *Sankhya.*
Wald Lecturer, 1974 Annual Meeting of the Institute of Mathematical Statistics.
President, Institute of Mathematical Statistics, 1974-75.

Invited to Department of Mathematics, University of Maryland to deliver six lectures (September 1975) as part of their "Year in Probability and Statistics" program.
Chairman of Editorial Board of the IMS-University of Chicago Monograph Series, from April 1977.
Fellow, American Academy of Arts and Sciences, from 1986.
Outstanding Statistician of the Year, Chicago Chapter of the American Statistical Association, 1992.

Publications

(1)　"On a problem in the theory of k populations," *Ann. Math. Statist. 21* (1950), 362-375.

(2)　"The problem of the greater mean" (with H. Robbins), *Ann. Math. Statist. 21* (1950), 469-487.

(3)　"A property of the *t* statistic," *Sankhya 12* (1952), 78-88.

(4)　"Impartial decision rules and sufficient statistics" (with Leo A. Goodman), *Ann. Math. Statist. 23* (1952), 553-562.

(5)　"Sufficiency and statistical decision functions," *Ann. Math. Statist. 25* (1954), 423-462.

(6)　"Two comments on sufficiency and statistical decision functions" (with E. L. Lehmann), *Ann. Math. Statist. 26* (1955), 139-142.

(7)　"A characterization of sufficiency," *Ann. Math. Statist. 26* (1955), 286-293.

(8)　"Statistics and subfields," *Ann. Math. Statist. 26* (1955), 490-497.

(9)　"Measurable subspaces and subalgebras," *Proc. Amer. Math. Soc. 6* (1955), 565-570.

(10)　"The nonexistence of certain statistical procedures in non-parametric problems" (with L. J. Savage), *Ann. Math. Statist. 27* (1956), 1115-1122.

(11)　"On unbiased estimates of uniformly minimum *variance,"* *Sankhya 18* (1957), 211-224.

(12)　"Examples of inconsistency of maximum likelihood estimates," *Sankhya 20* (1958), 207-210.

(13)　"A note on the fundamental identity of sequential analysis," *Ann. Math. Statist. 29* (1958), 534-543.

(14)　"Some approximations to the binomial distribution function," *Ann. Math. Statist. 31* (1960), 43-54.

(15)　"Simultaneous comparison of the optimum and sign tests of a normal mean," *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling,* Stanford University Press, (1960), 79-88.

(16)　"Stochastic comparison of tests," *Ann. Math. Statist. 31* (1960), 276-295.

(17)　"On the asymptotic efficiency of tests and estimates," *Sankhya 22* (1960), 229-252.

(18)　"On deviations of the sample mean" (with R. R. Rao), *Ann. Math. Statist. 31* (1960), 1015-1027.

(19)　"On the number of distinct values in a large sample from an infinite discrete distribution," *Proc. Nat. Inst. Sciences, India, 26,* A (Supp. 11), (1960), 67-75.

(20)　"A representation of the joint distribution of *n* dichotomous items," *Studies in Item Analysis and Prediction,* H. Solomon, ed., Stanford University Press, (1961), 158-168.

(21)　"On classification based on responses to *n* dichotomous items," *Studies in Item Analysis and Prediction,* H. Solomon, ed., Stanford University Press, (1961), 169-176.

(22)　"Classification into two multivariate normal distributions with unequal covariances" (with T. W. Anderson), *Ann. Math. Statist. 33* (1962), 420-431.

(23)　"On Fisher's bound for asymptotic variances," *Ann. Math. Statist. 35* (1964), 1545-1552.

(24)　"An optimal property of the likelihood ratio statistic," *Proc. Fifth Berk. Symp. Math. Statist. Prob., 1,* (1965), 13-26.

(25)　"A note on quantiles in large samples," *Ann. Math. Statist. 37* (1966), 577-580.

(26)　"Rates of convergence of estimates and test statistics," *Ann. Math. Statist. 38* (1967), 303-324. (This was a Special Invited Address to the Institute of Mathematical Statistics.)

(27)　"Substitution in conditional expectation" (with P. J. Bickel), *Ann. Math. Statist. 39* (1968), 377-378.

(28)   "On conditional test levels in large samples" (with P. J. Bickel), *University of North Carolina Monograph Series in Probability and Statistics,* No. 3 (1970), 25-34.

(29)   "Some asymptotic properties of likelihood ratios on general sample spaces" (with M. Raghavachari), *Proc. Sixth Berk Symp. Math. Statist. Prob., 1* (1970), 129-152.

(30)   *Some Limit Theorems in Statistics.* NSF-CBMS Monograph, No. 4 (SIAM, 1971).

(31)   "Examples of inconsistency of the likelihood ratio statistic," *Sankhya 34* (1972), 81-84.

(32)   "A note on UMV estimates and ancillary statistics." *Contributions to Statistics* (Hajek Memorial Volume), Academia (Prague), 1979, 19-24.

(33)   "On large deviations of the sample mean in general vector spaces," (with S. L. Zabell), *Ann. Probability, 7* (1979), 587-621.

(34)   "Large deviations, tests, and estimates" (with J. C. Gupta and S. L. Zabell). *Asymptotic Theory of Statistical Tests and Estimation* (Hoeffding Volume), 1979, 33-67. Academic Press.

(35)   "Hodges Superefficiency," 1980. *Encyclopedia of Statistical Sciences,* Vol. 3 (F-H). John Wiley.

(36)   "On large deviations of maximum likelihood and related estimates." Tech. Report No. 121, Department of Statistics, University of Chicago, 1980.

(37)   "A note on the effective variance of a randomly stopped mean." *Statistics and Probability: Essays in Honor of C. R. Rao,* 1982, 39-43, North-Holland Publishing Co.

(38)   "Some further properties of the LR statistic in general sample spaces" (with T. K. Chandra and D. Lambert), 1982. *Proceedings of the Golden Jubilee Conference,* 1-19, Indian Statistical Institute, Calcutta.

(39)   "Large deviations of the maximum likelihood estimate in the Markov chain case." *Recent Advances in Statistics* 1983, 273-286. Academic Press.

(40)   "Distributional optimality and second-order efficiency of test statistics" (with J. C. Gupta), 1986. In *Adaptive Statistical Procedures and Related Topics,* Proceedings of a symposium in honor of H. Robbins, IMS Lecture Notes Monograph Series, Vol. 8, 315-331.