Addendum: Open Problems

Open problems abound. Here we merely give a representative sample. The problems fall into two categories—logic and analysis.

We begin with questions motivated by logic.

1. Our first problem is concerned with the following: What is the degree of difficulty of those analytic processes which have been proved to the computable? One topic of broad scope is to bring into analysis the whole complex of problems associated with $P = NP$ (cf. Cook [1971], Karp [1972], Friedman, Ko [1982], Friedman [1984], Ko [1983], Blum, Shub, Smale [to appear]). Thus we may ask which analytic processes are computable in polynomial time, polynomial space, exponential time, etc. In the same manner, we can ask about levels of difficulty within the Grzegorczyk hierarchy, or any other subrecursive hierarchy. Or we could fix our attention on the primitive recursive functions. There is no reason to believe that the answers to these questions will be automatic extensions of the general recursive case.
2. For processes proved to be noncomputable, we can also ask for fine structure—this time via the theory of degrees of unsolvability. Most of the noncomputability results in this book make use of an arbitrary recursively enumerable nonrecursive set. In fact, any recursively enumerable nonrecursive set—of any degree of unsolvability—will do. The question is: Can we replace results which merely assert that a certain process is noncomputable by a fine structure for that process, involving different degrees of unsolvability?
3. Our third problem is concerned with nonclassical reasoning. We recall that the reasoning in this book is classical—i.e. the reasoning used in everyday mathematical research. This contrasts with the intuitionist approach (e.g. of Brouwer), the constructivist approach (e.g. of Bishop), and the Russian school (e.g. Markov and Šanin). A natural question is: What are the analogs, within these various modes of reasoning, of the results in this book?

   In this connection, we cite the work of Feferman [1984], who originated the system T_0 for representing Bishop-style constructive mathematics. T_0 has both constructive and classical models. In particular, Feferman reformulated our First Main Theorem in T_0, and left as an open question the status of our Second Main Theorem.
4. Our fourth problem concerns higher order recursion theory. Let us set the stage.

   Higher order recursion theory, of course, deals with functionals of functions from $\mathbb{N}$ to $\mathbb{N}$, functionals of such functionals, etc. A functional approach to recur-