Chapter XIII
E-Recursively Enumerable Degrees

Degree theory for subsets of *E*-closed structures differs markedly from the Σ_1 admissible case. On the surface the results are similar, but the modes of argument differ considerably. Post's problem once again has a positive solution, but this time without injuries and without repeated attempts to satisfy a given requirement. The presence of Moschovakis witnesses makes all the difference. Injuries do occur in the proof of Slaman's splitting theorem.

1. Regular Sets

Let $\mathscr{E}$ be transitive and $A, B \subseteq \mathscr{E}$. The relativization of *E*-recursiveness to *B* was introduced in Section 5.XI. In essence a new scheme,

$$\{c\}^B(x_1, \ldots, x_n) = B \cap x_i \quad (c = \langle 7, n, i \rangle)$$

is added to the original six. f is partial *E*-recursive relative to *B* if $f \simeq \{e\}^B$ for some $e < \omega$. *D* is *E*-recursively enumerable in *p* relative to *B* if

$$D = \{x \mid \{e\}^B(x, p)\downarrow\}$$

for some *e*. $\mathscr{E}$ is *E*-closed relative to *B* if

$$\{e\}^B(x)\downarrow \rightarrow \{e\}^B(x) \in \mathscr{E}$$

for all $e < \omega$ and $x \in \mathscr{E}$.

Assume $\mathscr{E}$ is *E*-closed. *A* is *E-reducible* to *B* on $\mathscr{E}$ (in symbols $A \leq_{\mathscr{E}} B$) if there exist $e < \omega$ and $p \in \mathscr{E}$ such that

(i) $\qquad \{e\}^B(x, p)\downarrow \quad$ for all $\quad x \in \mathscr{E}$,

(ii) $\qquad T_{\langle e, x, p; B \rangle} \in \mathscr{E} \quad$ for all $\quad x \in \mathscr{E}$, and

(iii) $\qquad A = \hat{x}[x \in \mathscr{E} \quad \& \quad \{e\}^B(x, p) = 0]$.