Bibliographical Remarks and Further Reading

Preliminaries

We try to indicate for each important result or notion its author and the corresponding publication, and possibly also reference to another work where the result is presented. Our aim is to be as precise as possible; on the other hand, these remarks are not intended to be a complete historical source and they serve only for orientation. This concerns mainly remarks on old results. The reader interested in deeper investigation of origins of the metamathematics of arithmetic is referred to source books: Gödel's Collected Works I [Gödel 86], the books From Frege to Gödel [van Heijenort 87] and The Undecidable [Davis 65]. We find also [Meschkowski 81] very informative. [Smoryński 91 – Logical] contains rather detailed historical information.

The material covered in the Preliminaries belongs to classics and can be found in standard monographs on mathematical logic, notably [Shoenfield 67], [Mendelson 64], [Bell-Machover 77] and others; note that the syntax of first order predicate logic is systematically developed in [Hilbert-Ackermann 28], largely in the style which is still in use. They formulate the problem of completness; Gödel presents his solution in his dissertation (1930) published as [Gödel 31 – Monatsh.]. [Skolem 20] contains a proof of what we now call the downward Löwenheim-Skolem theorem, using what we call Skolemization. The above works seem to deal with validity and satisfiability as intuitively clear notions without trying to formalize them. Tarski's paper ([Tarski 33] in Polish; a German version is [Tarski 36], for an English version see [Tarski 56]) presents conditions defining the satisfaction of a compound formula from the satisfaction of its components – these are our "Tarski's truths conditions" or "Tarski's satisfaction conditions" (for a detailed analysis of Tarski's approach see [Hodges 85]).

Herbrand's theorem is contained in his thesis published as [Herbrand 30]; as commented in [van Heijenort 67] p. 526, Herbrand considered his theorem to be a more precise statement of the well-known Löwenheim-Skolem theorem. From the above mentioned monographs, only [Shoenfield 67] elaborates on Herbrand's theorem.

Origins of first order arithmetic were described in the Introduction; let us add some details. Arithmetical hierarchy was introduced in [Kleene 43] and [Mostowski 47]; the notation Σ_n, Π_n, Δ_n is due to [Addison 58] and [Mostowski 59] (cf. Kleene's introductions in [Gödel 86]). Definition of functions from other functions by primitive recursion were known to Dedekind, Skolem, Hilbert and Ackermann. Gödel introduces in [Gödel 31 – Monatsh.] a class of functions that he calls recursive; in our present terminology, these are just primitive recursive functions. For origins of general recursive functions see [Péter 34], [Kleene 36]; monographs on recursive functions and recursion theory include [Rogers 67], [Soare 87]. Gödel introduces and uses coding of finite sequences of natural numbers by natural numbers [Gödel 31] and arithmetization of syntax. We shall comment more on arithmetization in remarks on Chaps. I and III.