Introduction

People have been interested in natural numbers since forever. The ancient mathematicians knew and used the principle of *descente infinie,* which is a form of mathematical induction. The principle is as follows: if you want to show that no number has the property φ, it suffices to show that for each number n having the property φ there is a smaller number $m < n$ having the property φ. (If there were a number having φ we could endlessly find smaller and smaller numbers having φ, which is absurd.) The Greeks used the principle for a proof of incommensurability of segments. The principle was rediscovered in modern times by P. Fermat (1601–1665). The principle of mathematical induction itself (if 0 has the property φ and for each number n having φ also $n+1$ has φ then all numbers have φ) seems to have been first used by B. Pascal (1623–1662) in a proof concerning his triangle. A general formulation appears in a work of J. Bernoulli (1654–1705). (Our source is [Meschkowski 78-81].)

In 1861 Grassman published his *Lehrbuch der Arithmetik;* in our terms, he defines integers as an ordered integrity domain in which each non-empty set of positive elements has a least element. In 1884 Frege's book *Grundlagen der Arithmetik* was published. We can say that Frege's natural numbers are classes; each such class consists of all sets of a certain fixed finite cardinality. (Frege speaks of concepts, not of classes.) The famous Dedekind's work *Was sind und was sollen die Zahlen* appears in 1888. Dedekind's natural numbers are defined as a set N together with an element $1 \in N$ a one-one mapping f of N into itself such that 1 is not in the range of f and N is the smallest set containing 1 and closed under f. Dedekind and Frege agreed that arithmetic is a part of logic, but differed in their opinions on what logic is. They both used the same main device: a one-one mapping and closedness under that mapping.

Dedekind was not interested in finding a formal deductive system for natural numbers; this was the main aim of Peano's investigation of natural numbers (*Arithmetices principia nova methoda exposita,* 1889). Peano's axiom system (taken over from Dedekind, who had it from Grassman) is, in our terminology, second order: it deals with numbers and sets of numbers. Nowadays