7. Selected Topics

As the title suggests this chapter is a collection of various more advanced topics. The first section, on bounded and unbounded theories, both contains useful facts about a natural class of theories and illustrates how the regular type machinery can be used to classify the models of a theory with relatively simple invariants. The second section delves more deeply into the properties of our notions of rank in some very special theories such as the uncountably categorical ones.

7.1 Bounded and Unbounded Theories

We work in a stable theory throughout the section.

Definition 7.1.1. *(i) The theory T is called* bounded *if there are $< |\mathfrak{C}|$ domination equivalence classes of nonalgebraic stationary types; T is* unbounded *if it is not bounded.*

(ii) The theory T is unidimensional *if any two nonalgebraic types are nonorthogonal.*

Shelah (and many others) call unbounded theories *multidimensional* and bounded theories *nonmultidimensional* or *nmd*, for short. There are many examples of such theories:

Lemma 7.1.1. *The theory of any infinite module is bounded.*

Proof. Let $\mathfrak{C}$ be the universal domain of the relevant theory. By Proposition 5.3.2 and Lemma 5.3.9 an element p of $S_1(\mathfrak{C})$ is the translate of the generic type in $stab(p)$, a group $\bigwedge-$definable over $\emptyset$. Certainly p is domination equivalent to this generic type. Since there are $\leq 2^{|T|}$ many such groups this is a bound on the number of domination equivalence classes. The same argument establishes this bound for types in other sorts (i.e., $n-$types in the $1-$sorted theory of the module), proving the lemma.

We will say little here about bounded theories which are properly stable. The superstable ones become easier to handle using

Lemma 7.1.2. *The following are equivalent for a superstable theory T:*