5. Stability

In this chapter we state and prove the basic definitions and theorems relevant to all stable theories. The first section contains the most fundamental material. Here a freeness relation (see Definition 3.3.1) called forking independence is developed which agrees with Morley rank independence on a t.t. theory. Many of the theorems proved earlier for t.t. theories can be generalized to stable theories, the class of theories on which forking independence exists.

Sections 5.1 to 5.3 contain material which anyone working in stable theories must know. The first-time reader should feel free to skip the proofs in Section 5.5, although it is important to know the statements of the results found there. The forking independence relation is analyzed more deeply in Section 5.6. A class of types (namely those having weight 1) is isolated on which a well-behaved dimension theory exists.

5.1 Stability

Here we define a broad class of theories (called the stable theories) on which there is a freeness relation satisfying the conditions specified in Definition 3.3.1. As with t.t. theories, the freeness relation is defined via a rank (more accurately, a family of ranks). Intuitively, each of these ranks could be described as "Morley rank relative to a finite set of formulas". The overall goal of the section is to develop the relevant ranks and notion of freeness, prove the definability of types in stable theories and relate its existence to the number of types over sets.

Remember: Every complete theory discussed is assumed to have built-in imaginaries.

5.1.1 Ranks and Definability

Writing the formula φ in the form $\varphi(\bar{x}, \bar{y})$ indicates that the free variables in φ are in $\bar{x}\bar{y}$, $\bar{x}$ should be regarded as a sequence of free variables in the usual sense, but $\bar{y}$ is a placeholder for a sequence of parameters. For example, a general quadratic polynomial in x can be written as $\varphi(x, abc) = ax^2 + bx + c$,