3. Uncountably Categorical and $\aleph_0$–stable Theories

In this chapter we will study the results which laid the foundation for stability theory, namely Morley's Categoricity Theorem and the Baldwin-Lachlan Theorem. Some of the concepts arising in their proofs will be redeveloped later for stable theories. We feel, however, that these proofs present an excellent introduction to the key concepts encountered later, and are historically important enough to warrant individual treatment.

In Section 1 a proof of Morley's Categoricity Theorem is given. In the third section of the chapter totally transcendental theories, which arose in Morley's original proof of Morley's Categoricity Theorem, will be studied more deeply. Again, ideas will be introduced which are seen throughout stability theory. In the fourth section these new concepts are applied to prove the Baldwin-Lachlan Theorem. Groups definable in totally transcendental theories are studied in the fifth section.

3.1 Morley's Categoricity Theorem

Throughout this section an arbitrary theory is assumed to be countable and have infinite models. For emphasis this assumption may be repeated within the statements of theorems. Recall that a theory T is said to be *categorical in* κ (or κ*–categorical*), where κ is an infinite cardinal, if T has a unique model of cardinality κ, up to isomorphism. A theory is called *uncountably categorical* if it is categorical in every uncountable cardinality. In the previous chapter theories were exhibited which are:

– categorical in every infinite cardinal;
– categorical in $\aleph_0$ but not in any uncountable cardinal;
– categorical in every uncountable cardinal, but not in $\aleph_0$;
– not categorical in any infinite cardinal.

It was conjectured by Łoś that every countable complete theory satisfies one of these four possibilities. Morley proved this conjecture with

Theorem 3.1.1 (Morley's Categoricity Theorem). *If a countable complete theory T is categorical in some uncountable cardinality then it is categorical in every uncountable cardinality.*