1. The Basics

1.1 Preliminaries and Notation

We assume that the reader is familiar with the basic definitions and results normally found in a first course in mathematical logic. Specifically, we will freely use the concepts of a *first-order language*, a *structure* or *model* in that language, and the *satisfaction relation* between models and formulas. We also assume that the reader knows the Compactness and Omitting Types Theorems, and can carry out an elimination of quantifiers argument for a specific theory such as dense linear orders without endpoints or divisible abelian groups. In this first section we will review some of these results as a way of setting our notation and viewpoint and jogging the student's memory.

Notation. (Model Theory)

- A first-order language is denoted by L, L', L_0, etc. The cardinality of a language L, $|L|$, is simply the cardinality of the set of nonlogical symbols of L.
- Formulas are denoted by lower case Greek letters. Writing $\varphi(v_0, \ldots, v_n)$ indicates that the free variables in φ are in $\{v_0, \ldots, v_n\}$. If $t_0, \ldots, t_n$ are terms in the language, $\varphi(t_0, \ldots, t_n)$ is the formula obtained by substituting t_i for v_i. A sentence is a formula with no free variables.
- We use $\mathcal{M}$ or $\mathcal{N}$, decorated with various subscripts and superscripts, to denote a model or structure in a first-order language. The universe of, e.g., $\mathcal{M}_0$, is M_0. Elements of the universe are denoted by lower case letters such as a, b, c, etc. If X is an element of the language in which $\mathcal{M}$ is a model $X^{\mathcal{M}}$ denotes the interpretation of X in $\mathcal{M}$.
- Given models $\mathcal{M}$ and $\mathcal{N}$ in a language L, a function $f : M \longrightarrow N$ is an *isomorphism* if f is a bijection and for all symbols $X \in L$, $f(X^{\mathcal{M}}) = X^{\mathcal{N}}$. When there is an isomorphism of $\mathcal{M}$ onto $\mathcal{N}$ we write $\mathcal{M} \cong \mathcal{N}$ and say $\mathcal{M}$ and $\mathcal{N}$ are isomorphic. An *automorphism* of $\mathcal{M}$ is an isomorphism of $\mathcal{M}$ onto itself. For $\mathcal{M}$ a model, $\mathrm{Aut}(\mathcal{M})$ denotes the automorphism group of $\mathcal{M}$.
- A *theory* in the language L is a consistent set of sentences of L. A set of sentences need not be complete in order to be called a theory. (A set of sentences is *consistent* if it has a model. A theory is *complete* if all