XIV. Iterated Forcing with Uncountable Support

§0. Introduction

This chapter is [Sh:250], revised. Here we consider revised support for the not necessarily countable case. In §1 we define and present the basic properties of κ-RS iterated. This includes the case $\kappa = \aleph_1$ and so it can serve as a substitute to X §1. The main difference is that here we have to use names which sometimes have no value as we cannot use rank as there.

Unlike Chapter X, we do not have a useful properness to generalize, so naturally the generalizations of completeness are in the center. In 2.1 we introduce, and in 2.4 we show it does not matter much if we use the version with games of length $\kappa = \mathrm{cf}\kappa$ or the version with a side order $\leq_0$, the "pure" extension which is κ-complete. Then we define iterations of such forcing notions and prove the basic properties (2.5–2.8). This repeats §1, so against dullness this time we waive the associativity law and simplify somewhat the definition of the iteration. In the definition of the order except finitely many places (which are names) the extensions are pure (i.e. $\leq_0$) in the old places. The first use of "pure" extensions is Prikry [Pr], and the first use of iterations with the distinction between old and new places (in normal support of course) is Gitik [Gi] which uses Easton support iteration $\bar{Q}$'s for high inaccessibles, each Q_i is $(\{2\}, \kappa_i)$-complete where for the important i's $\kappa_i = i$; a subsequent proof more similar to our case is [Sh:276, §1]. The application we have in mind is