I. Forcing, Basic Facts

§0. Introduction

In this chapter we start by introducing forcing and state the most important theorems on it (done in §1); we do not prove them as we want to put the stress on applying them. Then we give two basic proofs:
in §2, we show why CH (the continuum hypothesis) is consistent with ZFC, and in §3 why it is independent of ZFC. For this the $\aleph_1$-completeness and c.c.c.(=countable chain conditions) are used, both implying the forcing does not collapse $\aleph_1$ the later implying the forcing collapse no cardinal. In §4 we compute exactly $2^{\aleph_0}$ in the forcing from §3 (in §3 we prove just $V[G] \models$ "$2^{\aleph_0} \geq \lambda$"; we also explain what is a "Cohen real"). In §5 we explain canonical names.

Lastly in §6 we give more basic examples of forcing: random reals, forcing diamonds. The content of this chapter is classical, see on history e.g. [J]. (Except §7, 7.3 is A. Ostaszewski [Os] and 7.4 is from [Sh:98, §5], note that later Baumgartner has found a proof without collapsing and further works are:

P. Komjáth [Ko1], continuing the proof in [Sh:98] proved it consistent to have MA for countable partial orderings $+\neg$CH, and $\clubsuit$. Then S. Fuchino, S. Shelah and L. Soukup [FShS:544] proved the same, without collapsing $\aleph_1$ and M.Džamonja and S.Shelah [DjSh:604] prove that $\clubsuit$ is consistent with SH (no Souslin tree, hence $\neg$CH).)