

Introduction

Even though the title of this book is related to the old Lecture Notes “Proper Forcing”, it is a new book: not only did it have six new chapters and some sections moved in (and the thirteenth chapter moved out to the author’s book on cardinal arithmetic), but the old material and also the new have been revised for clarification and corrected several times.

Now, twenty years after its discovery, I feel that perhaps “proper forcing” is a household concept in set theory, and the reader probably knows the basic facts about forcing and proper forcing. However we demand no prerequisites except some knowledge of naive set theory (including stationary sets, Fodor Lemma, strongly inaccessible, Mahlo, weakly compact etc.; occasionally we mention some large cardinals in their combinatorial definitions (measurable, supercompact), things like $0^\#$ and complementary theorems showing some large cardinals are necessary, but ignorance in those directions will not hamper the reader), and the book aims at giving a complete presentation of the theory of proper and improper forcing from its beginning avoiding the metamathematical considerations; in particular no previous knowledge of forcing is demanded (though the forcing theorem is stated and explained, not proved). This is the main reason for not just publishing the additional material in a shorter book. Another reason is the complaints about shortcomings of the Proper Forcing Lecture Notes.

Forcing was founded by Cohen’s proof of the independence of the continuum hypothesis; Solovay and many others developed the theory (works prior to 1977). Particularly relevant to this book are Solovay and Tennenbaum [ST]