Appendix

Nonstandard Compactness Arguments and the Admissible Cover

One of the subjects we have not touched on in this book is applications of infinitary logic to constructing models of set theory and the relationship between compactness and forcing arguments. At one time we planned to include a chapter on these matters, but the book developed along other lines.

In this appendix we present one example of such a result because it leads very naturally to *the admissible cover* of a model $\mathfrak{M}$ of set theory. We want to treat this admissible set for two reasons. In the first place, it gives an example of an admissible set with urelements which has no counterpart in the theory without urelements, and it is as different from $\mathbb{HYP}_{\mathfrak{M}}$ as possible. Secondly, we promised (in Barwise [1974]) to present the details of the construction of this admissible set in this book.

1. Compactness Arguments over Standard Models of Set Theory

Let $\mathbb{A} = \langle A, \in \rangle$ be a countable transitive model of ZF. Then $\mathbb{A}$ is an admissible set and, moreover, $(\mathbb{A}, R)$ is admissible for every definable relation R. We can therefore apply Completeness and Compactness to $\mathsf{L}_{\mathbb{A}}$ or $\mathsf{L}_{(\mathbb{A},R)}$, for any such R. There are many interesting results to be obtained in this way; we present one here and refer the reader to Barwise [1971], Barwise [1974], Friedman [1973], Krivine-MacAloon [1973], Suzuki-Wilmers [1973], and Wilmers [1973] for other examples. We also refer the reader to Keisler [1973] for connections with forcing.

The axiom $\mathrm{V}=\mathrm{L}$ asserts that every set is constructible.

1.1 Theorem. *Let $\mathbb{A}$ be a countable transitive model of* ZF. *There is an end extension $\mathfrak{B} = \langle B, E \rangle$ of $\mathbb{A}$ which is a model of* $\mathrm{ZF} + \mathrm{V} = \mathrm{L}$.

Proof. Let T be the theory of $\mathsf{L}_{\mathbb{A}}$ containing:

ZF.

The Infinitary diagram of $\mathbb{A}$.

We need to see that $T \cup \{\mathrm{V}=\mathrm{L}\}$ has a model. If not, then

$$T \models \mathrm{V} \neq \mathrm{L}$$