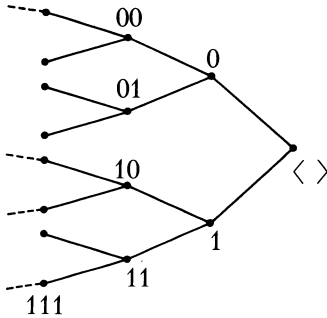# Chapter VIII
Strict Π_1^1 Predicates and König Principles

1. The König Infinity Lemma

In this section we discuss some of the uses of the Infinity Lemma in ordinary recursion theory. The applications chosen for discussion are those which become important new "axioms" or *König Principles*, when stated in the abstract.

Let $T = \langle T, \prec \rangle$ be the *full binary tree*, as pictured below.

The set T is the set of nodes (finite sequences of 0's and 1's) ordered by

$$d' \prec d$$

if the sequence d' properly extends the sequence d. If $S \subseteq T$ is such that $d_0 \in S$ and $d_0 \prec d_1$ implies $d_1 \in S$, then $S = \langle S, \prec \upharpoonright S \rangle$ is called a *subtree* of T. If S is a subtree then any maximal $\prec$-linearly ordered subset b of S is called a *branch through S*.

1.1 König Infinity Lemma. *Let $S = \langle S, \prec \upharpoonright S \rangle$ be any subtree of the full binary tree. The following are equivalent:*
  (i) *S has no infinite branch,*
 (ii) *S is well founded,*
(iii) *S is well founded and has finite rank,*
 (iv) *S is finite.*