Chapter III
Countable Fragments of $\mathsf{L}_{\infty\omega}$

In this chapter the student is introduced to the infinitary logic $L_{\infty\omega}$ and its countable fragments. The reason for treating infinitary logic so early in the book is two-fold. In the first place it offers a nice application of the very notion of admissible set, since the fragments of $L_{\infty\omega}$ most like ordinary logic are those given by countable admissible sets. More important, however, is the powerful tool that infinitary logic gives us in our study of admissible sets. The results from model theory presented in this chapter are all chosen because of their applicability to the theory of admissible sets and generalized recursion theory.

1. Formalizing Syntax and Semantics in KPU

In § I.3 we formalized informal notions of mathematics in KPU, notions like "function", "natural number", and "ordinal". In this section we do the same thing for informal notions of logic, notions like "language", "structure", "formula".

In this section we work in KPU but we suppose that among the atomic predicates of our metalanguage L^* are the following:

$$\begin{gathered}\text{Relation-symbol } (x),\\ \text{Function-symbol } (x),\\ \text{Constant-symbol } (x),\\ \text{Variable } (x)\end{gathered}$$

and among the operation symbols of our metalanguage are two unary ones:

$$\mathsf{v} \quad \text{and} \quad \# .$$

We use $\mathsf{r}, \mathsf{r}_1, \ldots$ to vary over objects x satisfying Relation-symbol (x). Similarly $\mathsf{h}, \mathsf{h}_1, \ldots$ for function symbols and $\mathsf{c}, \mathsf{c}_1, \mathsf{d}, \ldots$ for constant symbols. We also assume that among the constant symbols of our *metalanguage* L^* are

$$\neg,\ \bigwedge,\ \bigvee,\ \forall,\ \exists,\ \equiv .$$