Introduction

Since its beginnings in the early sixties, admissible set theory has become a major source of interaction between model theory, resursion theory and set theory. In fact, for the student of admissible sets the old boundaries between fields disappear as notions merge, techniques complement one another, analogies become equivalences, and results in one field lead to results in another. This is the view of admissible sets we hope to share with the reader of this book.

Model theory, recursion theory and set theory all deal, in part, with problems of definability and set existence. *Definability theory* is (by definition) that part of mathematical logic which deals with such problems. The Craig Interpolation Theorem, Kleene's analysis of Δ_1^1 sets by means of the hyperarithmetic sets, Gödel's universe L of constructible sets and Shoenfield's Absoluteness Lemma are all major contributions to definability theory. The theory of admissible sets takes such apparently divergent results and makes them converge in a single coherent body of thought, one with ramifications for all parts of logic.

This book is written for the student who has taken a good first space year graduate course in logic. The specific material we presuppose can be summarized as follows. The student should understand the completeness, compactness and Löwenheim-Skolem theorems as well as the notion of elementary submodel. He should be familiar with the basic properties of recursive functions and recursively enumerable (hereinafter r.e.) sets. The student should have seen the development of intuitive set theory in some formal theory like ZF (Zermelo-Fraenkel set theory). His life will be more pleasant if he has some familiarity with the constructible sets before reading §§ II.5, 6 or V.4—8, but our treatment of constructible sets is self-contained.

A logical presentation of a reasonably advanced part of mathematics (which this book attempts to be) bears little relation to the historical development of that subject. This is particularly true of the theory of admissible sets with its complicated and rather sensitive history. On the other hand, a student is handicapped if he has no idea of the forces that figured in the development of his subject. Since the history of admissible sets is impossible to present here, we compromise by discussing how some of the older material fits into the current theory. We concentrate on those topics that are particularly relevant to this book. The prerequisites for understanding the introduction are rather greater than those for understanding the book itself.