Part F

Advanced Topics in Abstract Model Theory

Abstract model theory is the attempt to systematize the study of logics by studying the relationships between them and between various of their properties. The perspective taken in abstract model theory is discussed in Section 2 of Chapter I. The basic definitions and results of the subject were presented in Part A. Other results are scattered throughout the book. This final part of the book is devoted to more advanced topics in abstract model theory.

Chapter XVII views part of our experience with concrete logics in an abstract light. A concrete logic is presented by describing a class of structures, telling how the formulas are built up, and how formulas are interpreted in structures. Since formulas can be viewed as well-founded trees, they can be represented as set-theoretical objects. Similarly, structures are usually thought of as certain kinds of set-theoretical objects. Thus, we can think of a logic $\mathscr{L}$ as given by two predicates of sets: "x is a sentence of $\mathscr{L}$" and "the structure x satisfies the sentence y of $\mathscr{L}$." Chapter XVII deals with the following general problem: What can we say about the model-theoretic properties of $\mathscr{L}$ if we have information about how these predicates can be defined? Two forms of definitions are considered, implicit (Section 1) and explicit (the rest). The usual style of the inductive definition of truth is of the first kind, with its set-theoretical explanation being of the second kind.

When the inductive clauses for a logic $\mathscr{L}'$ can be written down in a logic $\mathscr{L}$, in a suitable precise sense, one says that $\mathscr{L}$ is *adequate to truth in* $\mathscr{L}'$. This gives a useful "effective" relation between logics which, in certain cases, agrees with the relation $\mathscr{L}' \leq_{\mathrm{RPC}} \mathscr{L}$, though not in general. Of special interest are logics which are adequate to truth in themselves.

On the explicit side, one may consider the complexity of the definition of a logic in terms of the Levy hierarchy of set-theoretic predicates, and in terms of the strength of the meta-theory T needed for the definitions. Particularly significant are the cases where the satisfaction relation for $\mathscr{L}$ is Δ_1 relative to a set theory T, which is the same as its being *absolute* relative to models of T. This insures that the meaning of a sentence is not sensitive to which universe of set theory is being considered. Absoluteness has a number of applications to the characterization of the infinitary logics $\mathscr{L}_{\infty\omega}$, $\mathscr{L}_{\infty G}$, and $\mathscr{L}_{\infty V}$ discussed in Chapters VIII and X. The discussions of the implicit and explicit approaches in this chapter are largely independent.