

Chapter II

Extended Logics: The General Framework

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The contents of this chapter are intended to serve as preparation for the more specific or more advanced topics of the chapters that follow. We will pay equal attention to general notions and concrete systems. The first part of the material is concerned with basic notions and examples. In Section 1 we define general logical systems. Section 2 contains a description of numerous concrete examples together with an elaboration of their essential properties—as far as this can be given without greater effort. Section 3 is concerned with elementary and projective classes as a tool to compare the expressive power of logical systems. Applications include the systematic use of PC-reducibility for compactness proofs. In Section 4 numerous preceding examples are systematized by the notion of the Lindström quantifier, and an analogue of the Ehrenfeucht–Fraïssé characterization of elementary equivalence for logics with monotone quantifiers is proved. The second part of the chapter is concerned with a more systematic representation of central model-theoretical notions, divided into three groups around compactness (Section 5), Löwenheim–Skolem phenomena (Section 6) and interpolation (Section 7).

We assume that the reader is acquainted with basic notions and facts of first-order model theory. In general we will consider only one-sorted structures; however, since in some cases many-sortedness leads to a methodological enrichment even for one-sorted model theory (see, for instance, Examples 7.1.2), we give the definitions for the many-sorted case (provided the many-sorted formulation is not too tedious and is of practical value). If not stated otherwise, examples, results and proofs refer to the one-sorted version. In most cases it is not hard to give the many-sorted extensions. For example, this can be done by reduction to the one-sorted version using additional predicates (“Unification of Domains”, see Feferman [1968a, p. 13]). However, there are exceptions and the warning following Definition 2.1.1 should be consulted.

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