Part A

Introduction, Basic Theory and Examples

This part of the book provides a basic setting for the chapters that follow, by isolating examples and concepts that have emerged as central and by presenting some of the more basic methods and results. Chapter I discusses how the subject of model-theoretic logics got started, both the parts that have to do with extended logics, and the part having to do with abstract model theory. The chapter presupposes familiarity with only the most basic parts of first-order model theory, its syntax and semantics.

In Chapter II the basic concept of a logic is presented, with many examples, as well as the concepts of elementary and projective class and compactness, Löwenheim–Skolem and definability properties. The notion of one logic being stronger than another is introduced and studied. Examples discussed include higher-order logics, logics with cardinality and cofinality quantifiers, infinitary logics and other logics with generalized quantifiers and logical operations.

Given any particular logic $\mathscr{L}$ one central problem is that of understanding when two structures are $\mathscr{L}$-equivalent, that is, satisfy the same $\mathscr{L}$-sentences. Among the basic results of Chapter II is a characterization of $\mathscr{L}$-equivalence in terms of partial isomorphisms, for a wide range of $\mathscr{L}$. Here we have a good example of a method borrowed from first-order logic which really comes into its own only in the more general setting. Another important method presented in Chapter II is the use of projective classes (PC) for establishing countable compactness and recursive axiomatizability for a host of logics.

Chapter III begins with an exposition of Lindstrom's theorem, which shows that first-order logic is the strongest logic (of ordinary structures) which satisfies the compactness and Löwenheim–Skolem properties. First-order logic is also shown to be maximal with respect to other combinations of familiar properties. The methods used are those of partial isomorphisms and projective classes.

Lindstrom's theorem has become a paradigm for characterizing other logics. Among those discussed in Chapter III are certain infinitary logics and logics with added quantifiers. Chapter III ends with an abstract characterization theorem which covers Lindstrom's theorem as well as logics for other types of structures, like topological structures. This connects with work in Chapter XV.

These chapters are meant to be accessible to anyone with a knowledge of basic model theory for first-order logic. They provide the reader with the basic notions and viewpoint needed to appreciate what follows.