Epilogue

In contrast with those of a work of fiction, the lives of the characters of this book extend well beyond its covers. For the reader who has a yen to follow them a bit further, we discuss here briefly some of the more recent developments and offer some suggestions for further reading. In many cases the material is still very much in a state of development and we are attempting not to give a thorough and orderly presentation but rather to convey some of the flavor of current research. Some of the notational conventions of the preceding chapters do not apply here.

We begin with Inductive Definability, which has been either on stage or just in the wings in almost every section of the book. One of the most elementary questions which was not answered in the text is that of the closure ordinals and sets for most classes of non-monotone inductive operators over ω. To state these results concisely, we introduce some notation. For any class X of relations, let

$$|X| = \sup{}^{+}\{|\Gamma| : \Gamma \text{ is an inductive operator and } \Gamma \in X\};$$

$$X\text{-Ind} = \{R : R \text{ is reducible to } \bar{\Gamma} \text{ for some inductive operator } \Gamma \in X\};$$

$$X\text{-Hyp} = \{R : \text{both } R \text{ and } {\sim}R \text{ belong to } X\text{-Ind}\}.$$

Replacing "inductive" by "monotone" yields the definitions of $|X\text{-mon}|$, X-mon-Ind, and X-mon-Hyp.

For comparison we state first some of our earlier results in this notation:

(1) $|\Sigma^0_1| = \omega$; (III.3.3)

$\Sigma^0_1\text{-mon-Ind} = \Sigma^0_1$; (III.3.5)

$\Delta^0_{(\omega)} \subsetneqq \Sigma^0_1\text{-Ind} \subsetneqq \Delta^1_1$; (III.3.6, 7)

(2) $|\Pi^0_1| = |\Pi^0_1\text{-mon}| = |\Pi^1_1\text{-mon}| = \omega_1$; (IV.2.15, 16, 21)

$\Pi^0_1\text{-Ind} = \Pi^0_1\text{-mon-Ind} = \Pi^1_1\text{-mon-Ind} = \Pi^1_1$; (III.3.1, 2, IV.2.17)

(3) $|\Sigma^1_1\text{-mon}| = \omega_1[\mathsf{E}^{\#}_1]$;

$\Sigma^1_1\text{-mon-Ind} = \{R : R \text{ is semi-recursive in } \mathsf{E}^{\#}_1\}$; (VI.6.14)