Appendix B
Lattice Tables and Representation Theorems

Lattice tables and usl tables of various kinds were an important part of the proofs of Parts B and C which characterized various initial segments of $\mathscr{D}$. We now indicate how to construct such tables. These tables are related to representations of lattices as lattices of equivalence relations.

1. Finite Distributive Lattices

We construct lattice tables for finite distributive lattices. These tables are the ones needed to obtain the results of Chap. VI.

1.1 Definition. A lattice $\mathscr{L} = \langle L, \leqslant, \vee, \wedge\rangle$ is *distributive* if the following conditions are satisfied for all a, b, $c \in L$:

(i) $\quad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$

(ii) $\quad a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c).$

Given a finite distributive lattice $\mathscr{L}$ with $k + 1$ elements, we wish to construct a *homogeneous lattice table* for $\mathscr{L}$. This table will consist of a set of $k + 1$-tuples of integers $< n$. We recall some definitions from Chap. VI.

1.2 Definition. Let Θ be a set of $k + 1$-tuples and let $\mathscr{L}$ be a lattice with $k + 1$ elements, $\{p_0, p_1, \ldots, p_k\}$. Let $\alpha, \beta \in \Theta$ and $i, j, m \leqslant k$ be given. We say that $\alpha \equiv_i \beta$ if $\alpha^{[i]} = \beta^{[i]}$, i.e., if α and β *agree on coordinate i.* If $p_i \vee p_j = p_m$, then we say that $\alpha \equiv_{i \vee j} \beta$ if $\alpha \equiv_i \beta$ and $\alpha \equiv_j \beta$. If $p_i \wedge p_j = p_m$, then we say that $\alpha \equiv_{i \wedge j} \beta$ if there is a finite sequence $\gamma_0, \ldots, \gamma_r$ of elements of Θ such that $\alpha = \gamma_0 \equiv_i \gamma_1 \equiv_j \gamma_2 \equiv_i \cdots \equiv_j \gamma_r = \beta$.

1.3 Definition. Let $n, k \in N$ and $\Theta \subseteq [0, n)^{k+1}$ be given. Let $\mathscr{L} = \langle L, \leqslant, \vee, \wedge\rangle$ be a lattice with elements $\{p_0, \ldots, p_k\}$ such that p_0 is the least element of $\mathscr{L}$ and p_k is the greatest element of $\mathscr{L}$. Then Θ is said to be a *finite homogeneous lattice table* for $\mathscr{L}$ if the following conditions are satisfied:

(i) $\quad \forall \alpha, \beta \in \Theta(\alpha \equiv_0 \beta).$