Chapter XII
Initial Segments of $\mathscr{D}[\mathbf{0}, \mathbf{0}']$

Having embedded minimal degrees below $\mathbf{0}'$, it is natural to try the embed other uppersemilattices as initial segments of $\mathscr{D}[\mathbf{0}, \mathbf{0}']$. We prove such embedding theorems in this chapter. In the first four sections, we present a detailed proof of the embeddability of an arbitrary finite lattice as an initial segment of $\mathscr{D}[\mathbf{0}, \mathbf{0}']$. Extensions of this result to other usls or to embeddings below degrees other than $\mathbf{0}'$ are discussed in Sec. 5. These results are applied to prove theorems about $\mathscr{D}$ and $\mathscr{D}[\mathbf{0}, \mathbf{0}']$.

1. Weakly Uniform Trees

Let $\mathscr{L}$ be a fixed finite lattice, with elements $0 = u_0, u_1, \ldots, u_n = 1$. Fix a weakly homogeneous sequential table Θ for $\mathscr{L}$ as in Appendix B.2. Θ is then the union of an increasing sequence $\Theta_0 \subseteq \Theta_1 \subseteq \cdots$ of finite sets of $n + 1$-tuples. Θ gives rise to a recursive function f defined by $f(k) = |\Theta_k|$ for all $k \in N$, and hence to the set of strings $\mathscr{S}_f = \{\sigma \in \mathscr{S}: \forall i \in N(\sigma(i) < f(i))\}$.

It is tempting to try to embed $\mathscr{L}$ as an initial segment of $\mathscr{D}[\mathbf{0}, \mathbf{0}']$ by combining the proofs of Theorems VII.4.1 and IX.2.1, and so, to use partial uniform trees to construct the desired initial segment. There are severe problems, however, in carrying out such a program. For suppose that an attempt is being made to construct a uniform (binary) e-splitting partial subtree of Id_2. Let us suppose that $T(\emptyset)$, $T(0)$ and $T(1)$ have been defined. The partial trees of Chap. IX now require

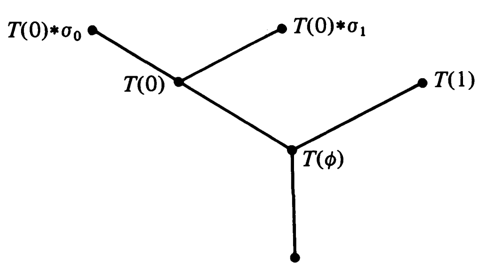

Fig. 1.1