Chapter X
Jumps of Minimal Degrees

Jump inversion theorems are used to characterize the range of the jump operator on various classes of degrees. In Chap. III, we proved two such theorems. The Friedberg Jump Inversion Theorem classified $\mathbf{J}[\mathbf{0}, \infty)$, the range of the jump operator on $\mathbf{D}[\mathbf{0}, \infty)$, as $\mathbf{D}[\mathbf{0}', \infty)$. And the Shoenfield Jump Inversion Theorem classified $\mathbf{J}[\mathbf{0}, \mathbf{0}']$, the range of the jump operator on $\mathbf{D}[\mathbf{0}, \mathbf{0}']$, as $\mathbf{D}[\mathbf{0}', \mathbf{0}^{(2)}] \cap \{\mathbf{d} : \mathbf{d}$ is recursively enumerable in $\mathbf{0}'\}$. This chapter is devoted to a proof of the Cooper Jump Inversion Theorem which classifies $\mathbf{J}(\mathbf{M})$, the range of the jump operator on the class of minimal degrees, as $\mathbf{D}[\mathbf{0}', \infty)$. This result contrasts sharply with the classification problem for $\mathbf{J}(\mathbf{M}[\mathbf{0}, \mathbf{0}'])$, the range of the jump operator on the class of minimal degrees below $\mathbf{0}'$, a problem which is still unsolved. The natural analogy would be to guess that $\mathbf{J}(\mathbf{M}[\mathbf{0}, \mathbf{0}']) = \mathbf{D}[\mathbf{0}', \mathbf{0}^{(2)}] \cap \{\mathbf{d} : \mathbf{d}$ is recursively enumerable in $\mathbf{0}'\}$. However, by IV.3.6, if $\mathbf{d} \in \mathbf{J}(\mathbf{M}[\mathbf{0}, \mathbf{0}'])$ then $\mathbf{d}' = \mathbf{0}^{(2)}$, so this guess is incorrect. Jockusch has conjectured that $\mathbf{J}(\mathbf{M}[\mathbf{0}, \mathbf{0}']) = \{\mathbf{d} : \mathbf{d} \geqslant \mathbf{0}' \,\&\, \mathbf{d}' = \mathbf{0}^{(2)} \,\&\, \mathbf{d}$ is recursively enumerable in $\mathbf{0}'\}$.

1. Targets

The strategy for proving the Cooper Jump Inversion Theorem is to combine the construction of a minimal degree using partial trees with the ideas introduced in the proof of the Friedberg Jump Inversion Theorem (III.4.2), making certain important modifications. One of these modifications involves defining a *jump target function*, which we do in this section. The proof of Cooper's theorem is presented in Sect. 2.

Given $C \subseteq N$, we build a set A such that $A' \equiv_T C \oplus \emptyset'$ as the union of a sequence $\{\alpha_s : s \in N\}$ of binary strings, through the use of an oracle of degree $\mathbf{0}'$. At stage e of the construction, we try to resolve whether or not $\Phi_e^A(e)\downarrow$. As, for $s > e$, α_s will be constrained to lie on certain partial recursive trees, we will not be able to ask an oracle of degree $\mathbf{0}'$ the same question as we asked in the proof of the Friedberg Jump Inversion Theorem. For with most reasonable recursively defined conditions, the search for a string which satisfies these conditions and which is not terminal on a given partial recursive tree requires an appeal to an oracle of degree $\mathbf{0}^{(2)}$. We therefore ask a different question, and insure that the answer to the new question at