Chapter XVIII

Vaught and Morley Conjectures for ω-Stable Countable Theories

In this chapter we complete the proof of two of the most important results about the spectrum problem. We prove Vaught's conjecture for countable ω-stable T: A countable ω-stable theory has either countably many or $2^{\aleph_0}$ countable models. Furthermore, we prove Morley's conjecture for countable ω-stable T: If $\aleph_1 \leq \kappa < \lambda$, $I(\kappa, AT) \leq I(\lambda, AT)$.

For Vaught's conjecture the assumption that T is ω-stable is made because it allows us to prove the theorem. The conjecture has not been resolved for any other class in the stability hierarchy although there are partial results by [Lascar 1981] and [Shelah 1978a]. In contrast, for Morley's conjecture the assumption that T is ω-stable is part of a systematic program. If T is not superstable then T has 2^λ models in power λ for all uncountable λ. Although we did not prove this result for singular λ in Section IX.6, we did prove the function was increasing for stable but not superstable T. Thus, the only omissions in our treatment of Morley's conjecture for countable theories are the unstable case and the superstable but not ω-stable case. The first of these cases is handled in the first edition of [Shelah 1978] while both are solved in the second edition.

For each problem we know by Section XVI.3 that T does not have the appropiate version of the dimensional order property. That is, if T has the DOP then T has 2^λ models of power λ for $\lambda > \aleph_0$. In addition, if the ω-stable theory T has the ENI-DOP then T has $2^{\aleph_0}$ countable models. In Chapter XVII, we completed the solution of the spectrum problem in uncountable cardinalities for ω-stable T except for shallow theories with finite depth. For theories with finite depth we did not resolve the difficulties which arise when there are ENI-types on the leaves of the representing tree of a model. The resolution of this difficulty is closely connected to the analysis of depth two types which is necessary to prove Vaught's conjecture for countable ω-stable T.

In Section 1 of this chapter we begin the analysis of types of low depth by considering what it means for one type to support another and some transfer properties of this notion. We justify this study of types with low depth in Section 2 by showing that if there is an ω-stable counterexample to