Chapter XVII

NDOP: Theories Without the Dimensional Order Property

In this chapter we investigate theories without the dimensional order property. As in the previous chapter, our main concerns will be with **S**-models of superstable theories and arbitrary models of ω-stable theories. We will show that for any acceptable class K, if T does not have the DOP, each K-model of T can be decomposed as a K-prime model over a skeleton which is an independent tree of 'small' K-models. If the model has cardinality λ, this tree will be isomorphic to a subset of $\lambda^{<\omega}$.

The following simple example is a good prototype for the kind of counting done in this chapter. The theory in the example is *deep* in the sense made precise in Section 2 and does not have the dimensional order property. Thus by Theorem 4.8, it has 2^λ models of power λ for each uncountable λ. We sketch here how this result can be seen directly.

Let T be the theory with a single function symbol f such that there is a unique point which is mapped to itself by f and all points have infinitely many preimages. T admits elimination of quantifiers. The models of T are best regarded as unions of components where two elements a and b are in the same component if for some m and n, $f^m(a) = f^n(b)$. There are, in fact, 2^λ possibilities for the unique component which has a base point so the others can be ignored in calculating the number of models of power λ. The component with a base point can be thought of as a subset of the tree $\lambda^{<\omega}$ which is closed under predecessor. Thus our task reduces to showing there are 2^λ such trees. To see this, we will show that each ordinal $\alpha < \lambda$ can be coded by a tree, $T_\alpha \subseteq \lambda^{<\omega}$. The T_α are constructed similarly to the coding of an arbitary countable ordinal by a subtree of $\omega^{<\omega}$. The difference is that rather than placing a single copy of T_β, for each $\beta < \alpha$, on the first level to code α, we place λ copies of each such T_β. This allows each point to have infinitely many preimages without disturbing the effect of the coding. There is a series of exercises at the end of Section 3 which explores the meaning of the notions defined in this chapter for this example.

Section 1 contains some preliminary results on the type of tree we use for a skeleton. In Section 2 we discuss representations of models and prove the decomposition theorem. In Section 3 we define the depth of a theory and