Chapter XVI

The Dimensional Order Property

We discuss in this chapter the main dividing line between superstable theories with and without a good structure theory; this dividing line is called, 'the dimensional order property' or just DOP. If a theory has the dimensional order property then it is possible to interpret an arbitrary binary relation into T by considering the dimensions of sets. This 'nonstructure' result leads to the conclusion that T has 2^λ models in every cardinality λ greater than $2^{|T|}$. The great significance of this concept stems from the perhaps even more remarkable consequences of its negation. Essentially, the negation of DOP (NDOP) is the assertion that the relation $p \dashv M$ is a trivial dependence relation. This hypothesis allows one to decompose every model as a tree of small models. There are a number of equivalent formulations of DOP which are useful in various contexts. The name is suggested by the following variant. If T has DOP then there exists a two parameter family $\{p_{\bar{a}\bar{b}}\}$ of copies of a type p such that for any choice of infinite cardinals $\lambda_{\bar{a},\bar{b}}$, there is a model M with $dim(p_{\bar{a}\bar{b}}, M) = \lambda_{\bar{a},\bar{b}}$. This leads to the construction of 2^κ models with cardinality $\kappa \geq 2^{|T|}$ by constructing $\bar{a}_i$, $\bar{b}_i$ for $i < \kappa$ and using $dim(p_{\bar{a}_i\bar{b}_j})$ to encode an arbitrary binary relation on κ.

In the first section of this chapter we discuss the notion of free amalgamation of models in a class K. This leads to the formalization of DOP as an assertion about the triviality of $\dashv$. We develop in Section 2 some technical properties of trivial types which are extremely useful in Section 3 and Chapter XVII. In Section 3, we show that the DOP implies T has many models. In the following chapter we will continue the discussion of theories without the DOP.

1. Avatars of the Dimensional Order Property

We first formally define K-NDOP in a way that will be useful in Chapter XVII and beyond. Then we show that the K in K-NDOP was superfluous; the dimensional order property does not, in fact, depend on the class K. We show that NDOP can be described in terms of finitely generated models