Chapter XV

The Width of a Theory

Our discussion of independence has focused on individual types. We have defined the dimension of a type p in a model M and if the type is regular or even has weight one then this dimension is always well defined. We have seen, however, that we may need to consider the dimension of more than one type in order to specify a model. In Section 1 of this chapter we define the width of a theory so that when this width is a cardinal number, it is the number of types whose individual dimensions must be specified to determine a model. Unfortunately, the situation is not always that simple.

There are three important cases. If all types have the same dimension, we call the theory unidimensional. Unidimensional theories are a natural generalization of $\aleph_1$-categorical theories. If there is a cardinal $\delta(T)$ such that we must specify the dimension of $\delta(T)$ types to determine each model, we call the theory bounded. If no such $\delta(T)$ exists, we call the theory unbounded. Section 2 begins the study of unbounded theories. We prove there lower bounds on the number of models for an unbounded theory. The bounded case is the comparatively straightforward generalization of vector space theory to allow several dimensions. In Section 3 of this chapter we derive a number of properties of the spectrum of a bounded theory.

The unbounded case is much more complicated. It is no longer possible to define a single unordered set as a basis for a model. Rather, there is a skeleton which is, roughly speaking, partially ordered by dependence. Chapter XVI is devoted to a detailed analysis of such skeletons.

We consider in Section 4 an extension of the notion of homogeneity to obtain some more detailed information about countable models of bounded theories. In particular, we solve Vaught's conjecture for ω-stable bounded theories.