Chapter XIII

Decomposition Theorems and Weight

In this chapter we will show every type in a superstable theory can be decomposed in terms of a finite number of regular types. In fact, under a suitable operation we impose a structure on the stationary types which reflects the multiplicative structure of the natural numbers. The weight one types behave as primes in this representation. We first obtain a precise structure theory for finitely generated extensions of an **S**-model. We connect these structural results with the notion of weight. While we would like to develop such a decomposition theorem for models in an arbitrary acceptable class K, we can not do so uniformly. Rather, we first obtain the result for **S**-models in Section 1 then define weight in Section 2. We conclude Section 2 by invoking the notion of weight to prove Lachlan's theorem that a countable superstable theory has either 1 or infinitely many countable models. In Section 3 we show that in an ω-stable theory there are 'enough' **AT**-strongly regular types. With this tool we obtain an extension of the decomposition theorem to all models of an ω-stable theory in Section 4.

Except for a few results at the beginning of Section 2, we assume in this chapter that T is superstable.

1. The Decomposition Theorem For S-Models

In this section we restrict ourselves to the class of **S**-models. We show that each finitely generated **S**-model of a superstable theory has a well defined dimension. We will use this information to decompose all types in a superstable theory as a product of regular types. The results of this section provide one characterization of weight. In Section 4, we consider extending the results of this section to other classes K.

1.1 Definition. Let M be an **S**-model, $A \subseteq M$, and let $R(M, A)$ be the collection of points in M which realize stationary regular types over A. Then $\dim(R(M, A))$ is the cardinality of a maximal independent subset of $R(M, A)$.