Chapter XII

Regular Types

In this chapter we introduce the most important concepts involved in assigning dimension to models. We saw in Section 1.3 that the simplest sort of $\aleph_1$-categorical theory is one in which the universe of each model is strongly minimal. A strongly minimal set D has two crucial features: i) its dimension is well-defined, ii) if $M \subseteq N$, every element of $D(N) - D(M)$ is independent from M over the empty set. The possession of both of these features is no accident; we will show that properly formulated versions of the two properties are equivalent. For any acceptable class K, we define the concept of a K-strongly regular type. The definition given in Section 1 is less intuitive but technically more useful than either of the properties just described. We then recast this definition in terms of the second of these properties. This recasting simplifies the construction of regular types in Section 2. In Section 3 we analyze invariance of dimension in terms of the transitivity properties of the forking relation. This approach makes it clear that for any K-strongly regular (stationary) type based on $A \subseteq M$, $\dim(p, M)$ is well defined. Finally, in Section 4 we show that the two approaches to defining regular types yield the same class of types. With this we can describe the relation between $\not\perp$ and $\triangleright^e$. Two types are orthogonal if and only if they are disjoint in the partial ordering imposed by domination. We further conclude that $\not\perp$ is an equivalence relation on the K-strongly regular types. Moreover, if $M \prec N$ and q is the nonforking extension of p to $S(N)$ then $\dim(p, N) = \dim(p, M) + \dim(q, N)$.

Throughout this chapter we assume that K is an acceptable class of models.

1. Weak Isolation and Regular Types

The usual notion of regular type is primarily associated with **S**-models and superstable theories; strongly regular types are similarly associated with arbitrary models of an ω-stable theory. We develop in this chapter a common framework for the two notions.