Part C

Local Dimension Theory

In Part A, we began the study of an independence relation for elements of models of stable theories. In Chapter II we noticed that, taking dependence to be the negation of independence, our notion does not in general satisfy the transitivity axiom for dependence. In Part B, we sought a global remedy for this problem. That is, we considered several notions of dependence which are transitive and thus provide a notion of the 'closure' of a set. Since none of these are the negation of the independence relation, they do not immediately yield a notion of dimension. In this part, we return to the study of nonforking and study those types, p, such that forking is transitive when restricted to realizations of p. We will be able to assign a well defined dimension, even in the finite case, to the set of realizations of such a 'regular' type.

There are, however, several complications. The most important is that we can not single out the 'regular' types with one definition which is appropriate for any theory T. Rather, we must define a notion of K-regularity where K is the set of **I**-saturated models for an isolation relation **I** and vary the choice of **I** according to the stability class of T.

Our goal is to provide a structure theory for the models of a theory T. As a test problem we try to calculate the spectrum function of T. In general, this problem is too difficult to be attacked directly. We can directly attack the problem of calculating the number of **S**-models of a superstable theory and the number of models of an ω-stable theory. The methods of attack on these two problems in [Shelah 1978], [Shelah 1982], and [Harrington & Makkai 1985] are parallel. Thus, in this book we have attempted to develop a single theory which specializes to the two cases. For this, we define in Chapter XI the notion of an acceptable class K. In so far as is possible, we provide a common development of the theory for any such class. In fact, the class of **S**-saturated models plays a distinguished role which prevents a completely uniform treatment. The formulation of the notion of acceptable class K given here and the general treatment of the concepts of regularity are only a first attempt at a theory which we hope can be extended eventually, e.g. to the infinitary case.

The bulk of Part C is devoted to the development of what is sometimes referred to as 'intermediate' stability theory. In Part D, we begin the study