Chapter X

Freeness and Isolation

In this chapter we investigate the relation between the notion of isolation (analogous to dependence) and the notion of freeness (analogous to independence). Since in fact we study only one notion of freeness (nonforking) but several notions of isolation, we could consider this study as the further investigation of properties of isolation relations. In fact, one of the notions of isolation, namely **S**-isolation, will be seen to play a rather privileged role. Of the dependence relations we consider, it is the most natural counterpart to the independence relation of nonforking. Unfortunately, study of **S**-isolation gives direct information about only the **S**-saturated structures. By making more restrictive assumptions on the theory we are able to widen the class of models treated. In particular, for ω-stable theories $\mathbf{AT}_{\aleph_0}$ will be covered by our discussion. In Section 1 we develop a set of axioms relating the notions of isolation and independence. Section 2 explores the notion of a 'powerful' isolation relation. In Section 3 we prove the uniqueness of prime models. Section 4 discusses the posssible sizes of indiscernible sequences over a set A which lie in prime models over A.

WARNING: While we have attempted to isolate in Chapter II all relevant features of nonforking, in Chapter IX the relevant features of isolation relations and here the properties which govern their interaction, attempts to apply these results to other isolation relations should be undertaken with caution. In particular, we have not carefully surveyed the situation for **F**-isolation. Our major purpose in this axiomatization has been to systematize the exposition of the properties of $\mathbf{AT}_\lambda$, $\mathbf{SET}_\lambda$, and $\mathbf{S}_\lambda$ for regular λ. We also include some material on **L**-isolation for countable languages.

1. Axioms Relating Freeness and Isolation

In this section we discuss four principles from which we derive all the information relating forking and the formation of prime models (subject to the caveat in the introduction to this chapter). The first of these makes the rather obvious assertion that a type which is isolated over a set A does not fork over A. The second is a transfer principle which asserts that if an