

Chapter VIII

Normalization and T^{eq}

In this chapter we give a brief introduction to some important topics in stability theory which do not play a major role in the part of stability theory emphasized in this book. These notions are extremely significant in the study of ω -stable, ω -categorical theories and their importance for the fine structure of the spectrum of models is becoming evident.

The first of these is the introduction of ‘imaginary elements’ to represent classes of equivalence relations which are definable in a theory T . This notion could have been used far more systematically in this book. Instead, we have placed a greater reliance on strong types. The second section is concerned with normalization, a topic which is closely related to the first and which leads to the introduction of geometric notions into the theory. We briefly discuss the role of these notions in the study of totally categorical theories in Section 3.

1. T^{eq}

The discussion of finite equivalence relations in Chapter IV suggests the desirability of giving special attention to the classes of definable equivalence relations. This idea is also suggested by the usual practice of forming quotient structures in algebra. Accordingly, we introduce an expanded language in which the equivalence classes of a definable equivalence relation can be considered as points. Some authors [Cherlin, Harrington, & Lachlan 1985] have proposed adding such equivalence relations piecemeal as they are needed. We follow Shelah in describing one large expansion, L^{eq} , which will encompass all the constructions we need to make. However, following a suggestion of Makkai we treat the expanded structure as a many-sorted model.

The properties of theories in this expanded language are ‘conservative’ for the non-technical results in stability theory. That is, T^{eq} provides a shortcut to prove theorems which would be much more difficult to conceptualize or prove without this notion. Shelah called the elements in the expanded models imaginary elements in analogy with the same conservative