Chapter VI

Orthogonality

In this chapter we investigate two extensions of the nonforking notion in an attempt to find the right notion of freeness. In the first section we discuss triples (A, B, C) such that A and B are not only independent over C but are persistently so. We say $t(A; C)$ is orthogonal to $t(B; C)$. In the second section we discuss a way in which A can be even more independent from B over C, namely $t(A; C)$ is orthogonal to every type over B. Our goal is this chapter is to describe the principal properties of these relations so we can use them as tools later. We first consider orthogonality of two types as a freeness relation in the sense of Chapter II. It turns out that, while regarding nonorthogonality of two types as a dependence relation is very natural; viewing nonorthogonality of a type and a set as a dependence relation is somewhat forced. A more natural approach to this latter relation is considered in Section 3 where we introduce an important partial (pre)order on the types over a set: the dominance order.

1. Orthogonality Of Types

In this section we discuss an important extension of the notion of independence: orthogonality. This concept has both a local and global form. The local form describes the orthogonality of two types over subsets of the monster model; the global form describes the orthogonality of two global types (i.e. types over the monster model). The notion of parallelism bridges the gap between these two forms of orthogonality. We begin by defining the local form.

1.1 Definition. Let p and q be complete types over C. We say p is *orthogonal* to q (over C) and write $p \perp q$ if the following holds. For every E containing C and every $\bar{a}$ realizing p, $\bar{b}$ realizing q, if $\bar{a} \downarrow_C E$ and $\bar{b} \downarrow_C E$ then $\bar{a} \downarrow_E \bar{b}$.

To simplify notation, we may write $(\bar{a} \perp \bar{b}; C)$. The role of C turns out to be unimportant; we usually just write $p \perp q$.